Mass-orbit modeling of galaxy clusters: mass profiles, orbital shapes by galaxy type & pseudo phase-space density

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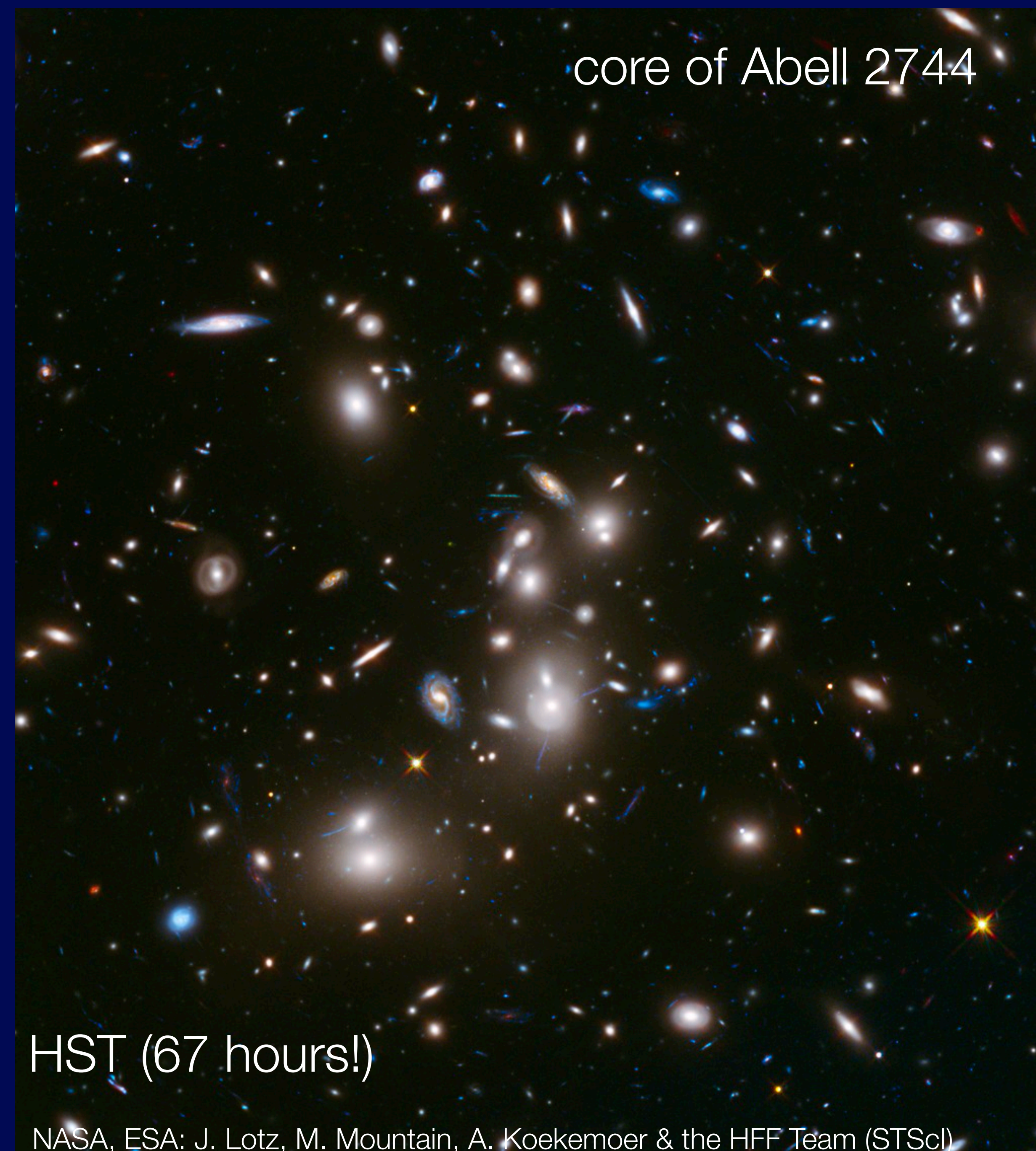
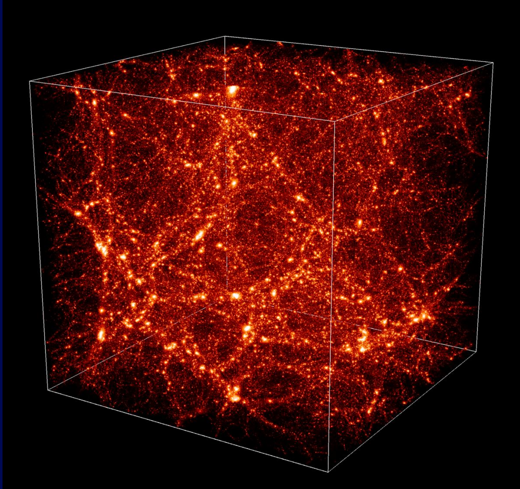
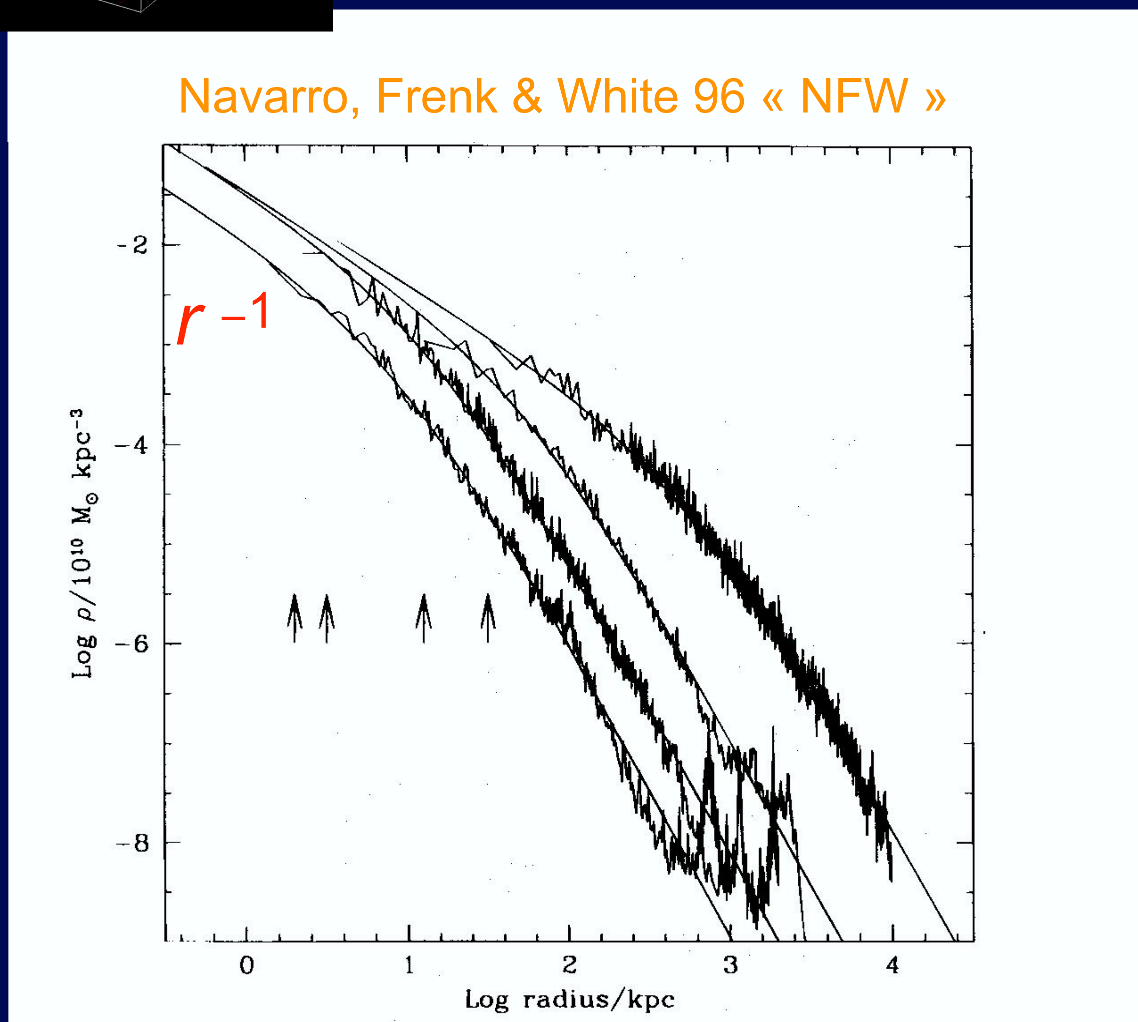
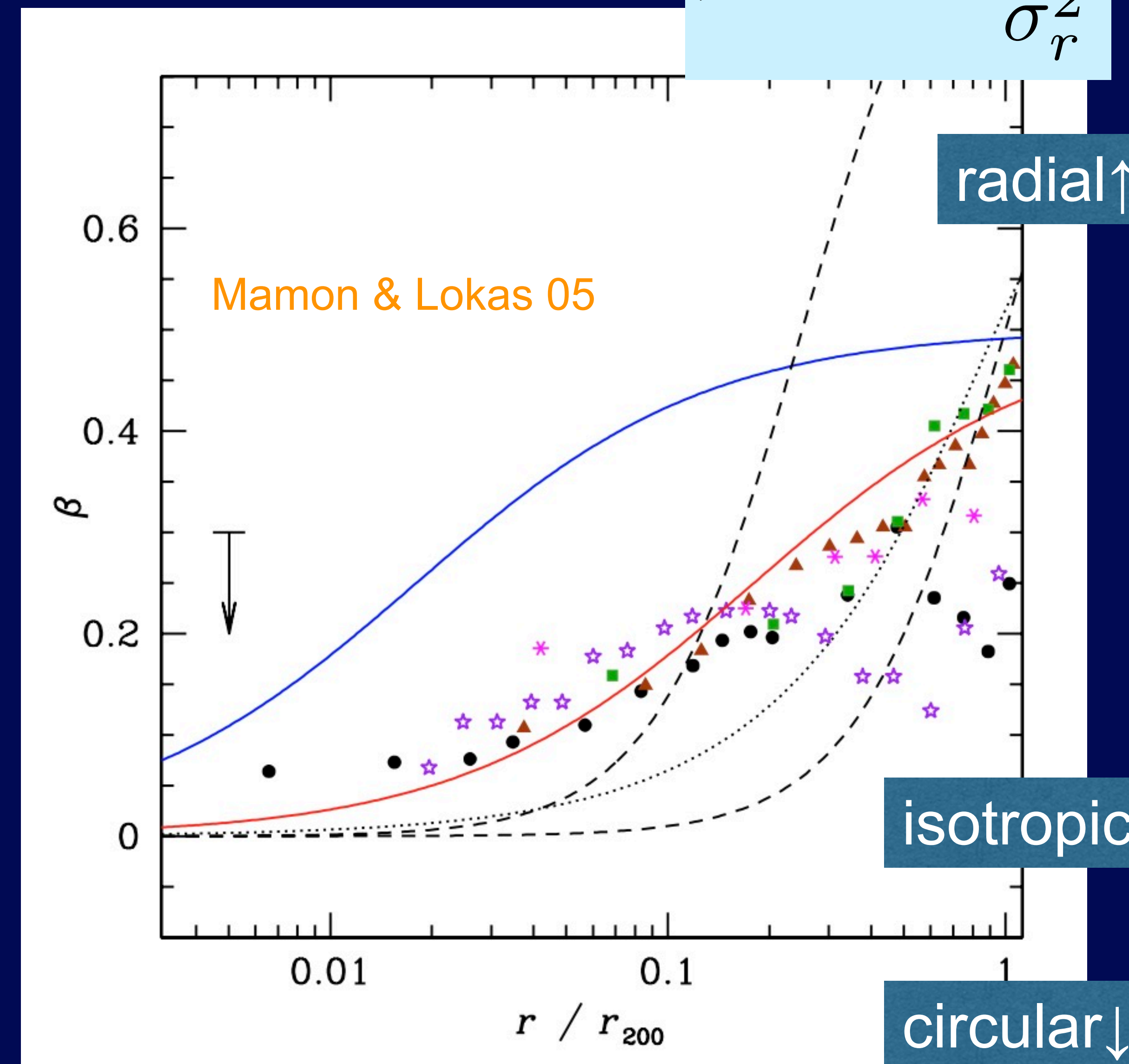


Dissipationless cosmological simulations



velocity anisotropy $\beta = 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$

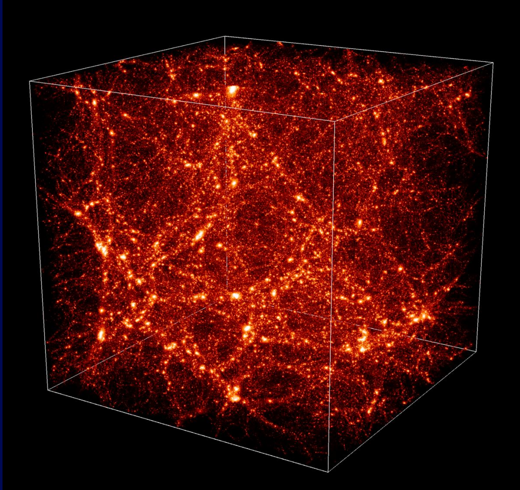




NFW-like density profiles

isotropic inner orbits
somewhat radial outer orbits

virial radius: radius of quasi-dynamical equilibrium
mean density ≈ 100x critical density of Universe



Dissipationless cosmological simulations

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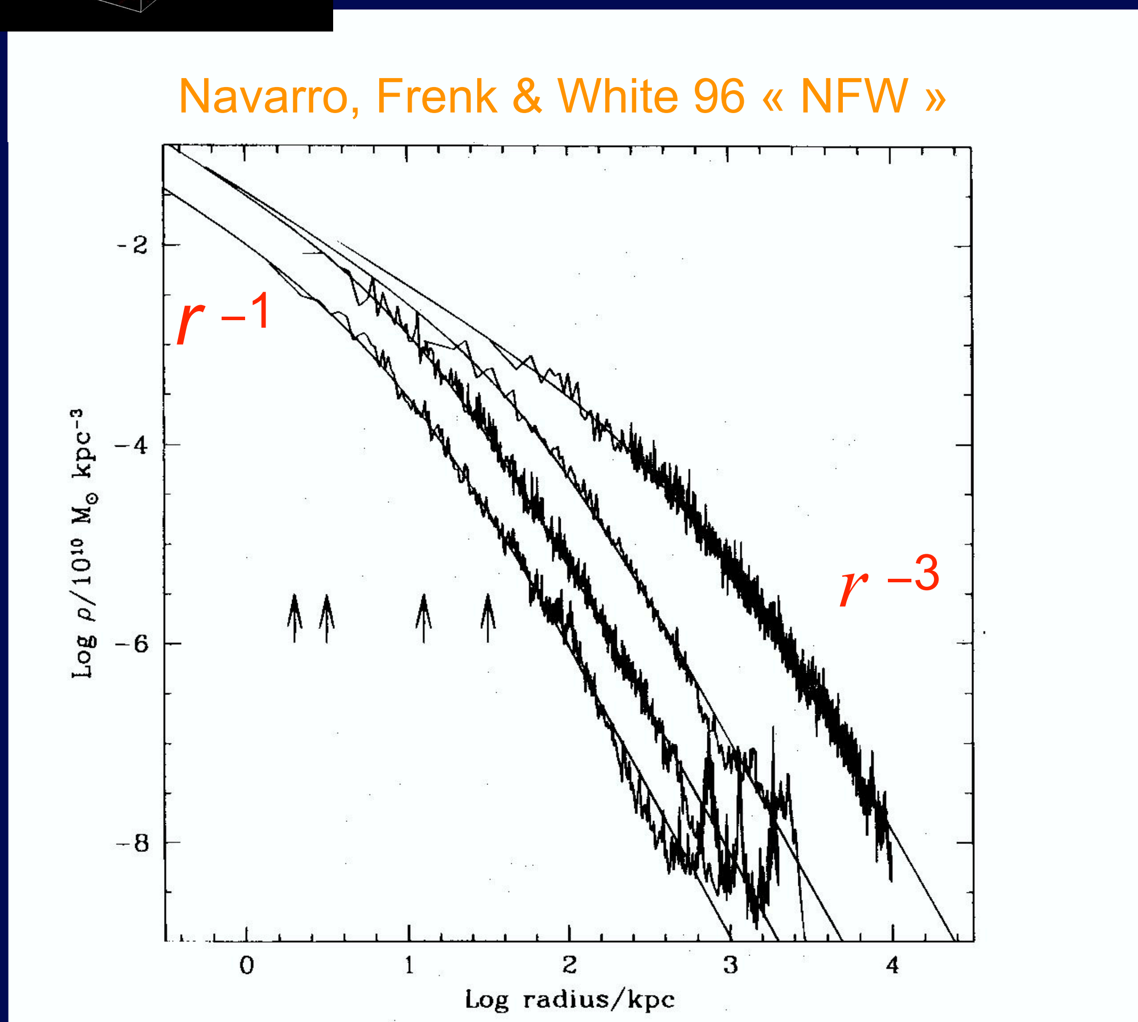
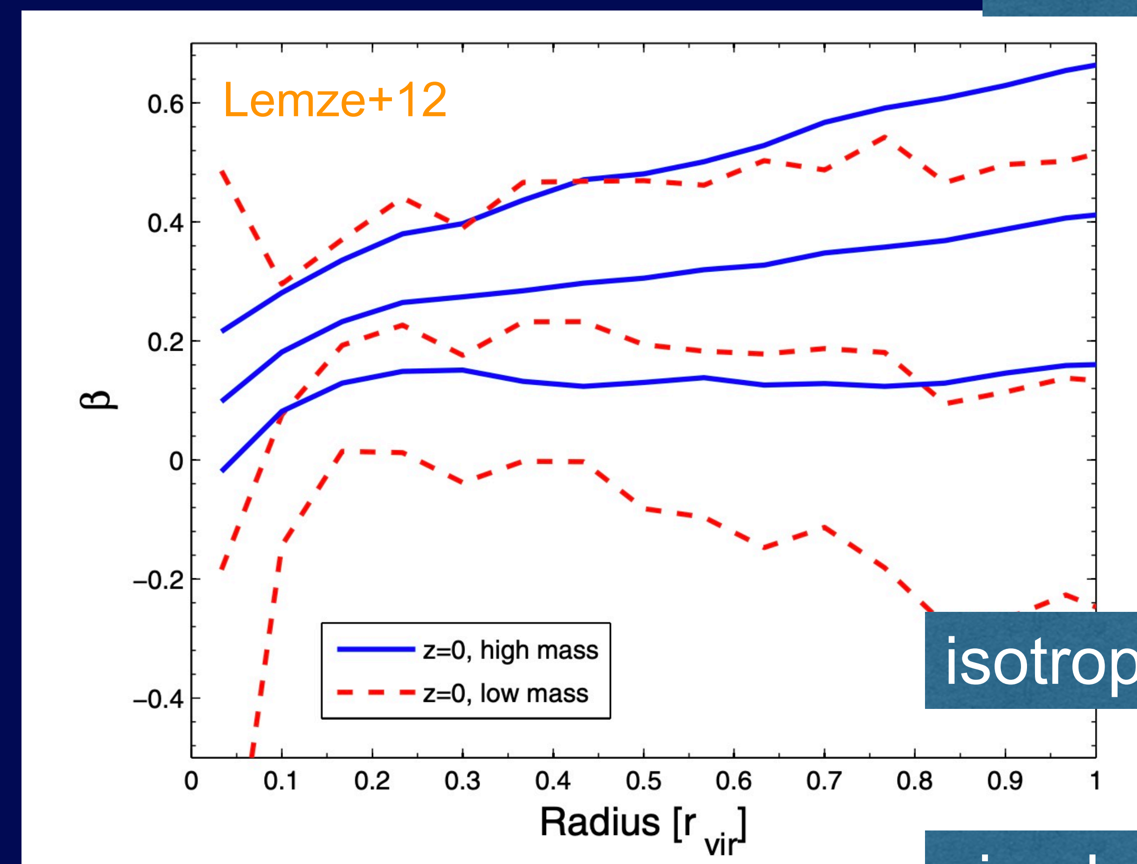


NFW-like density profiles

radial↑



isotropic

circular↓

isotropic inner orbits
somewhat radial outer orbits
esp. in hi-mass clusters

Cluster density profiles: concentrations

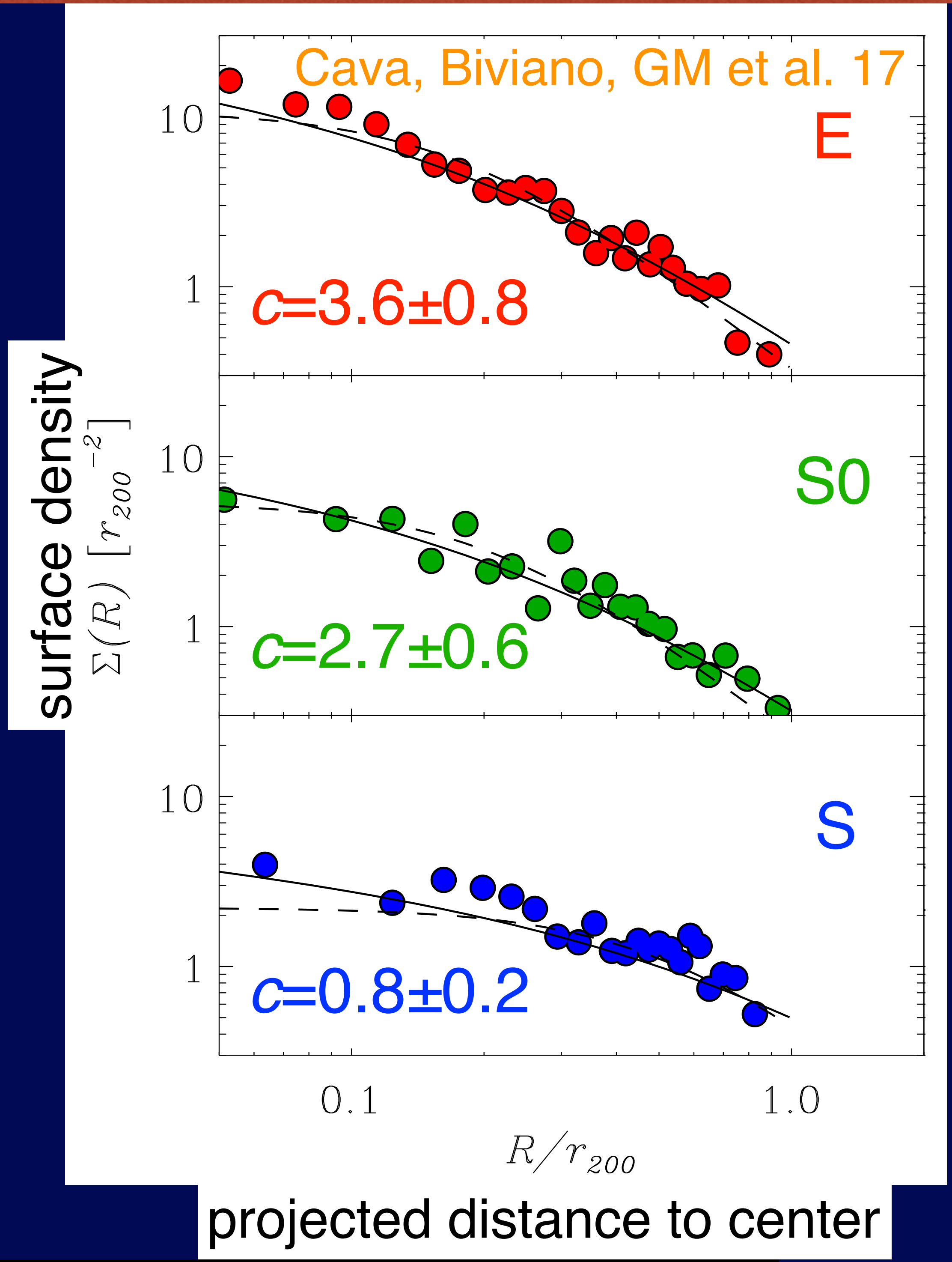
concentration $c = \frac{r_{\rm vir}}{r_{-2}}$

ΛCDM simulations: $c_{\rm mass} = 4$
Navarro+97

projected NFW fits well surface number density profile with $c = 4$ Carlberg+97

c(red) = 4, c(blue) = 1.3
Collister & Lahav 05

Stack of 54 z~0.05 regular (WINGS) clusters

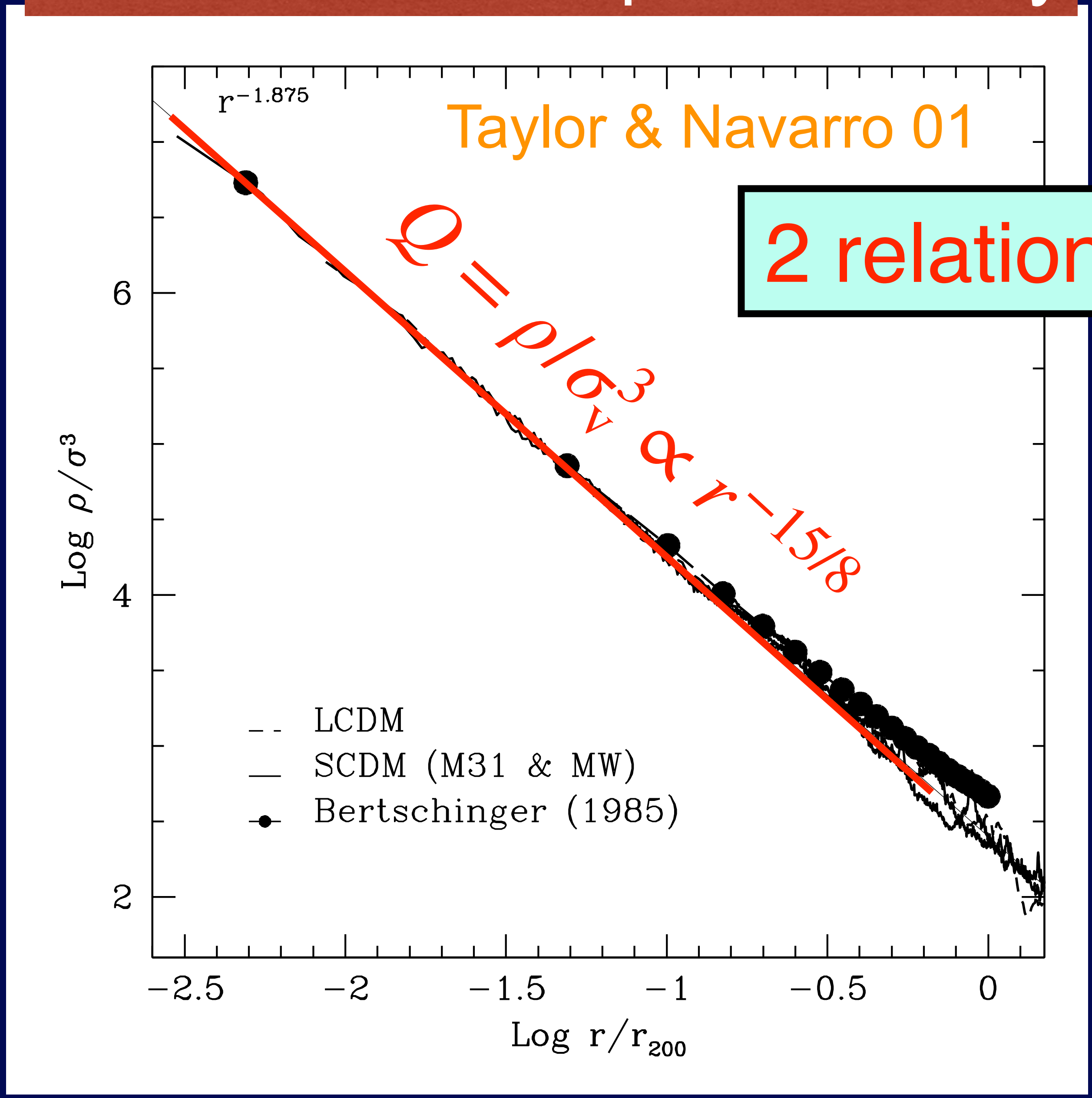


⇒ Ellipticals follow mass, spirals 4x wider distribution
S0s closer to ellipticals

Scaling relations from simulations

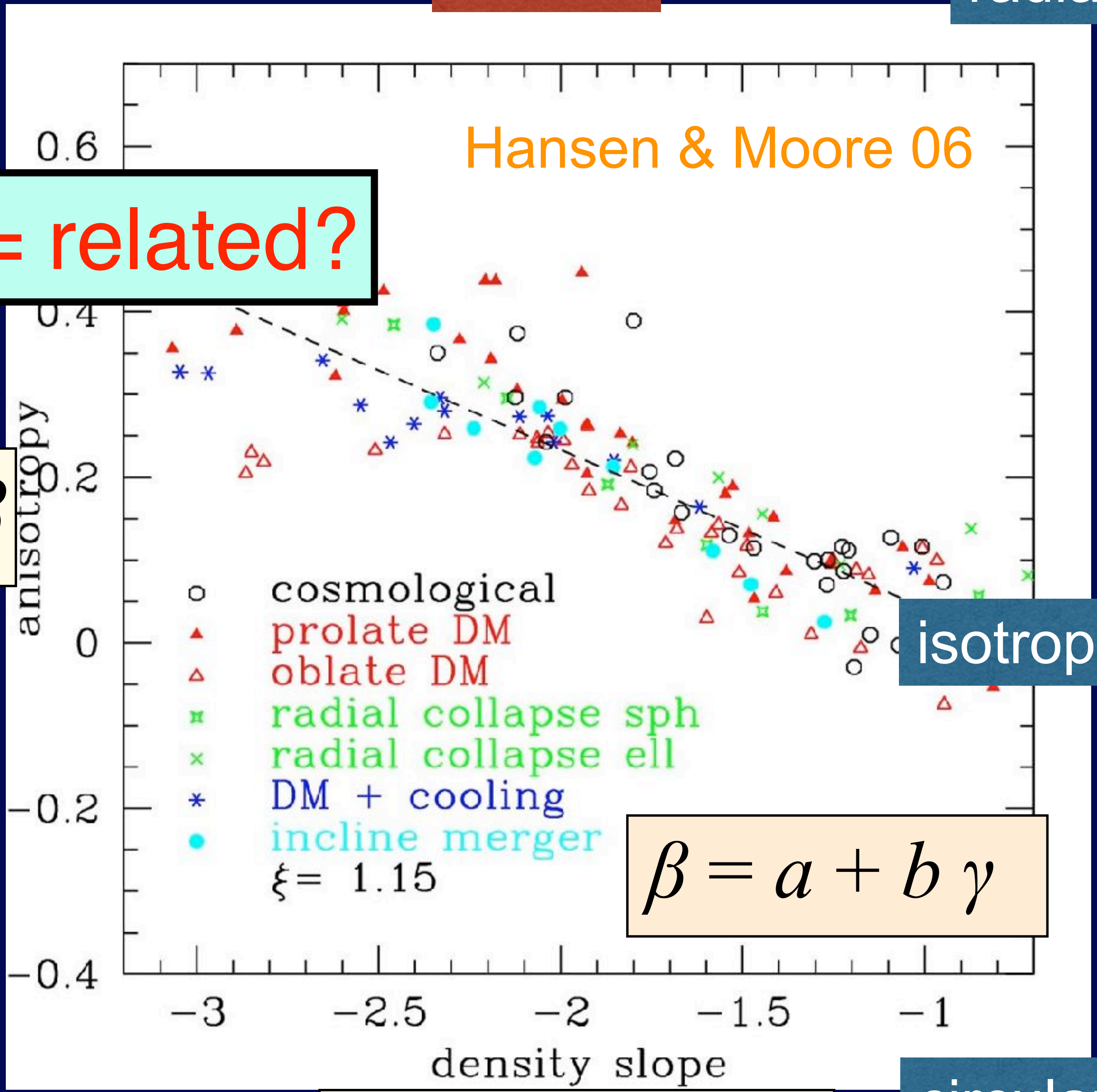
velocity anisotropy $\beta = 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$

Pseudo-Phase-Space-Density



2 relations = related?

$\beta - \gamma$



$\beta = a + b\,\gamma$

$\gamma = \mathrm{d}\ln\rho\,/\,\mathrm{d}\ln r$

Motivations: analyze observations of galaxy clusters to test & interpret what we learn from cosmological simulations of Dark Matter

Don't fully trust *hydrodynamical* simulations!

Dark Matter
- normalization
- shape
- concentration

↔ tests of standard cosmological model

Galaxy orbital shapes
- inner
- outer

↔ history of cluster buildup
constraints on galaxy formation
origin of S0 galaxies

Scaling relations
- density profiles
- concentration vs. mass
- pseudo phase space vs. radius
- velocity anisotropy vs. density slope

↔
- gravitational potential?
- dark matter?
- observational tracer?

Mass - orbit modeling method(s)

Measuring dark matter in spheroidal systems: using different physics

Newtonian dynamics: mass/orbit modeling

Jeans Equation

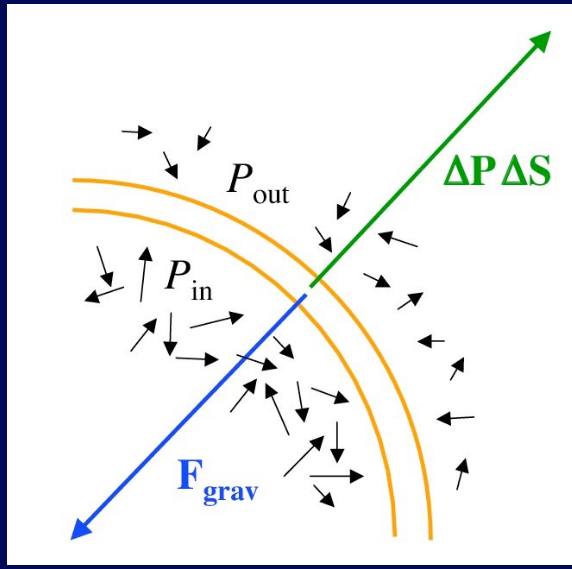
$$\nabla \cdot \mathbf{P} = -\nu \, \nabla \Phi$$

$$P_{ij} = \nu \, \langle v_i v_j \rangle$$

P = (anisotropic) pressure tensor

ν = tracer number density

Φ = potential



Hydrodynamics: hydrostatic equilibrium of X-ray gas

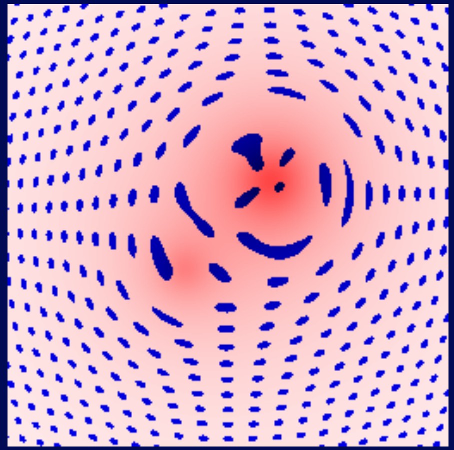
$$\nabla p = -\rho \, \nabla \Phi$$

p = pressure

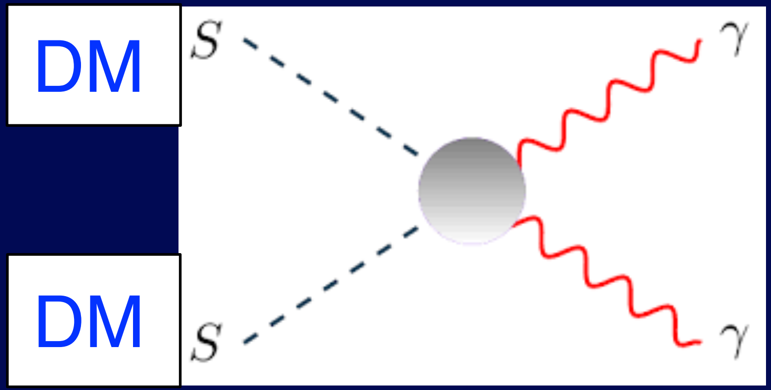
ρ = gas (mass or number) density

purple = hot gas

General relativity: gravitational lensing

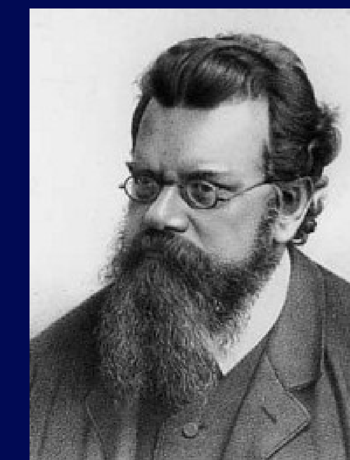


Particle physics: γ-ray annihilation or decay



From phase space to local space

$f = f(r,v) \equiv$ distribution function = 6D phase space density

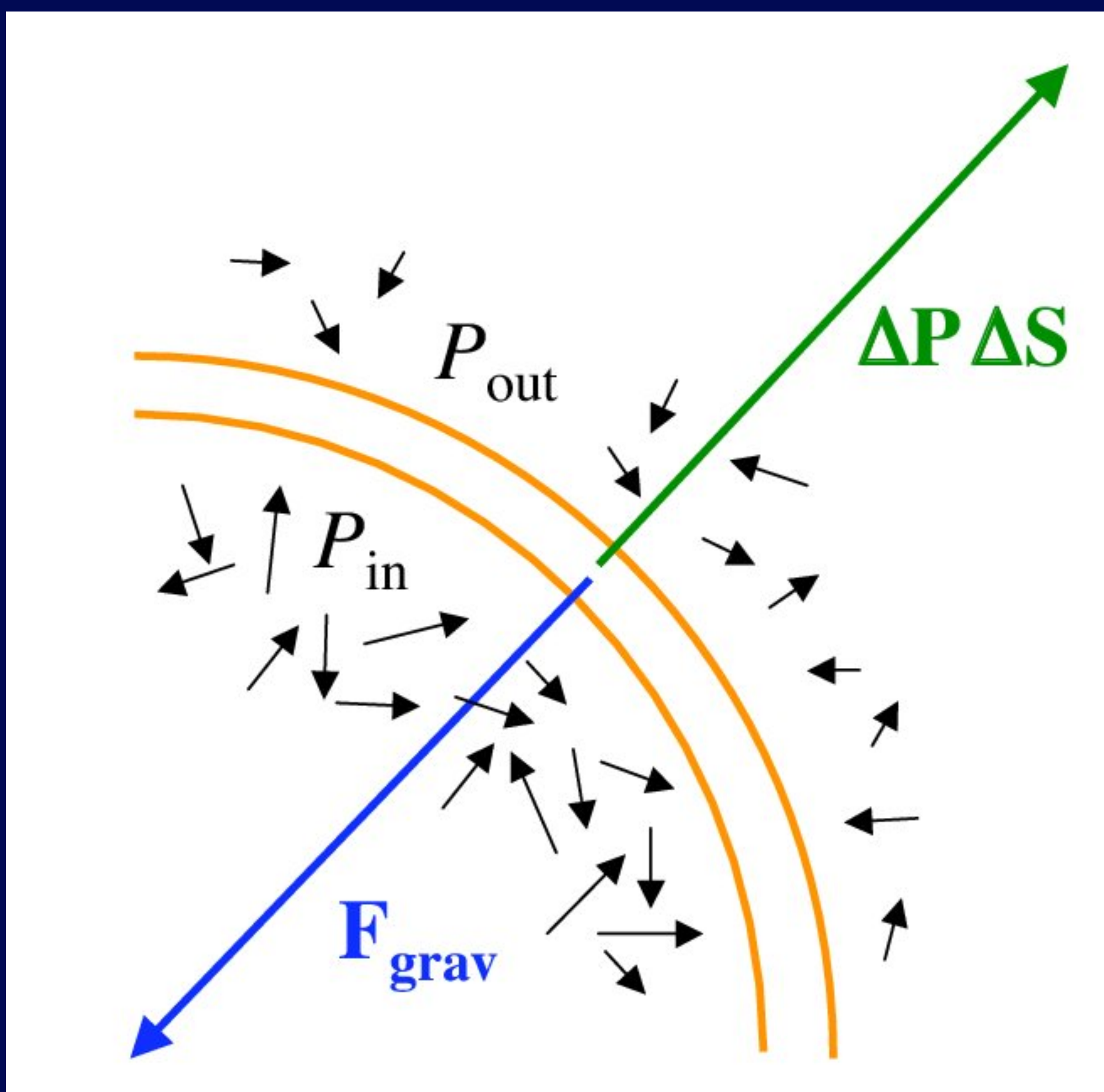


Collisionless Boltzmann Equation

incompressible 6D fluid

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla\Phi\cdot\frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\int v_j \,\mathrm{CBE}\, d^3\mathbf{v}$$

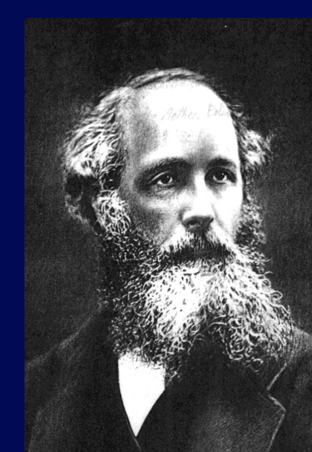


$$\nabla\cdot \boldsymbol{P} = -\nu\,\nabla\Phi$$

Jeans Equation

tracer density

$$\boldsymbol{P} = \nu\,\boldsymbol{\sigma}_v^2$$



Maxwell

Jeans

Spherical stationary Jeans equation

tracer density

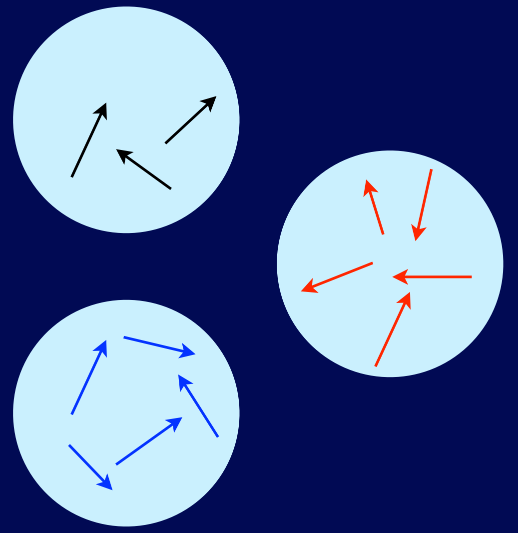
anisotropic dynamical pressure

$$\frac{d\left(\nu\sigma_r^2\right)}{dr} + 2\frac{\beta(r)}{r}\,\nu\sigma_r^2 = -\nu\frac{GM(r)}{r^2}$$

$$\beta(r) = 1 - \frac{\sigma_\theta^2(r)}{\sigma_r^2(r)}$$

= velocity anisotropy

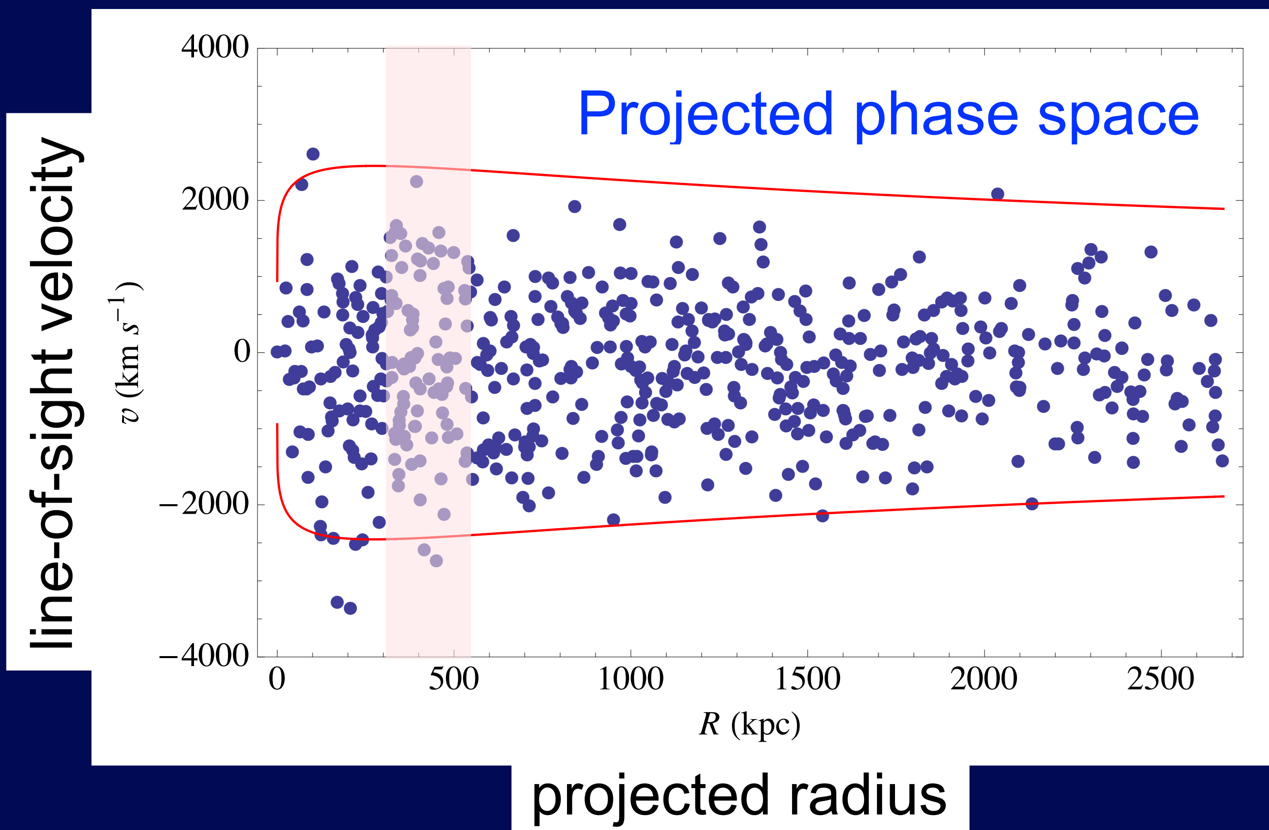
isotropic orbits: $\beta = 0$
radial orbits: $\beta = 1$
circular orbits: $\beta \to -\infty$

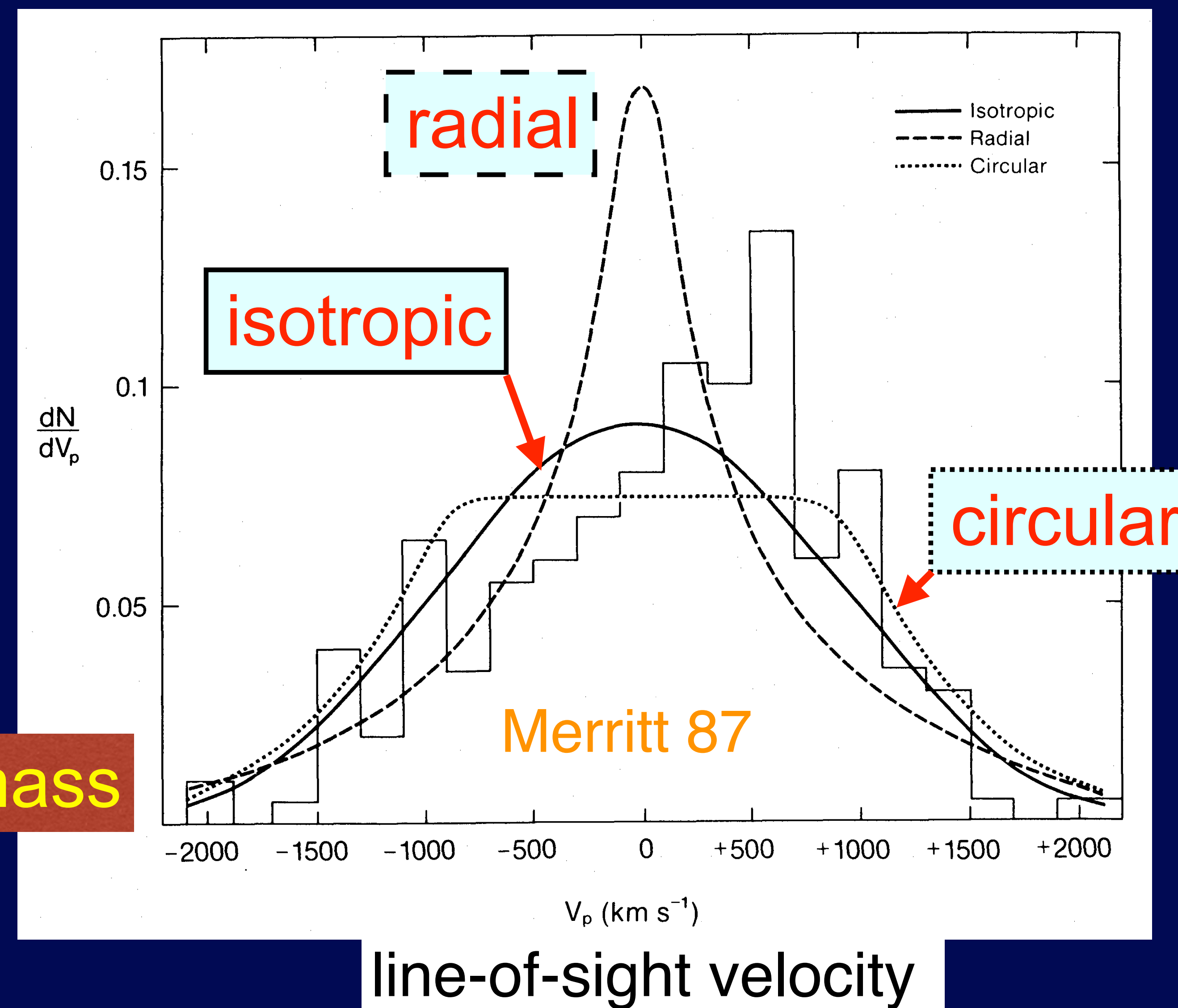


⇒ mass / anisotropy degeneracy

MAD

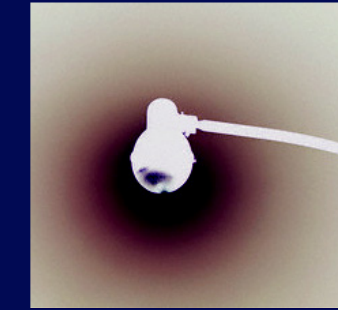
Diagnostics from *line-of-sight velocity distribution*





wider velocity distribution ⇒ more mass

cuspier velocity distribution ⇒ more radial orbits



MAMPOSSt: *Modeling Anisotropy & Mass Profiles*

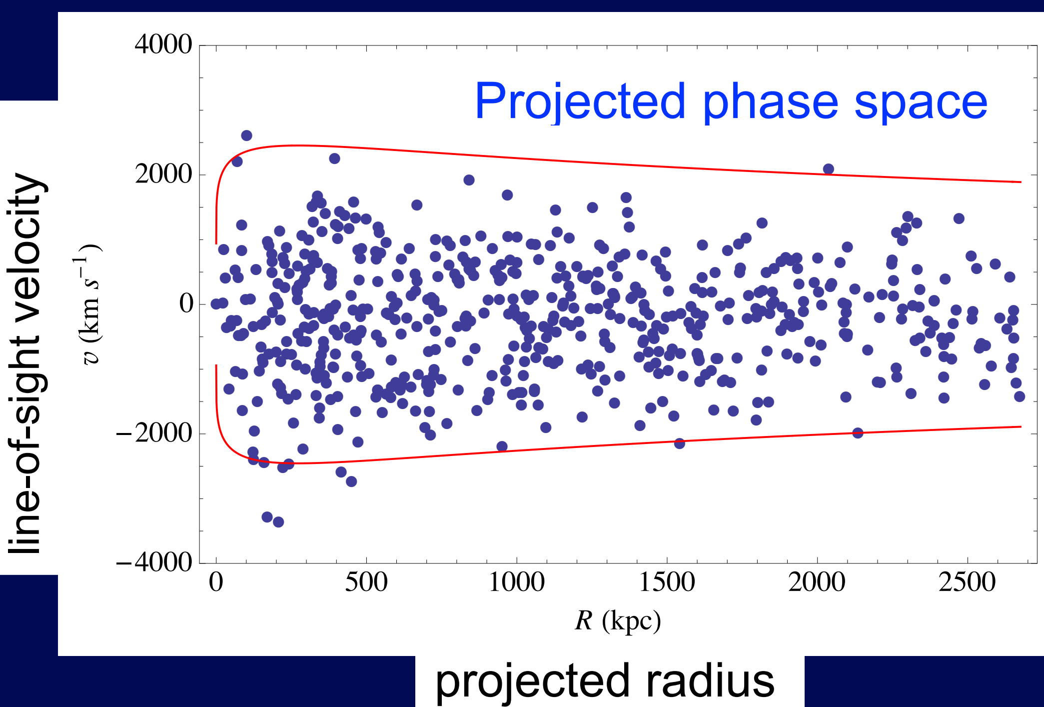
Mamon, Biviano & Boué 13

PDF of distribution in projected phase space

$$p(R, v_z) = \frac{4\pi R}{\Delta N_\mathrm{p}} \int_R^\infty \frac{r\,\nu(r)}{\sqrt{r^2 - R^2}}\, h(v_z|R, r)\,\mathrm{d}r$$

ν = tracer volume density

h = *local* velocity distribution function



Gaussian 3D velocities:

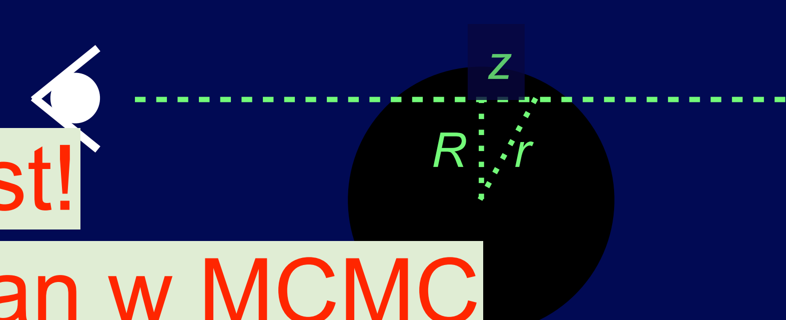
$$h(v_z|R, r) = \frac{1}{2\pi\sigma_z^2(R, r)} \exp\left[-\frac{v_z^2}{2\,\sigma_z^2(R, r)}\right]$$

$$\sigma_z(R, r) = \sqrt{1 - \beta(r)\left(\frac{R}{r}\right)^2}\,\sigma_r(r)$$

Binney & Mamon 82

solution to Jeans equation of local dynamical equilibrium

$$\sigma_r^2(r) = \frac{1}{\nu(r)} \int_r^\infty \exp\left[2\int_r^2 \beta(t)\frac{\mathrm{d}t}{t}\right] \nu(s)\frac{GM(s)}{s^2}\,\mathrm{d}s$$

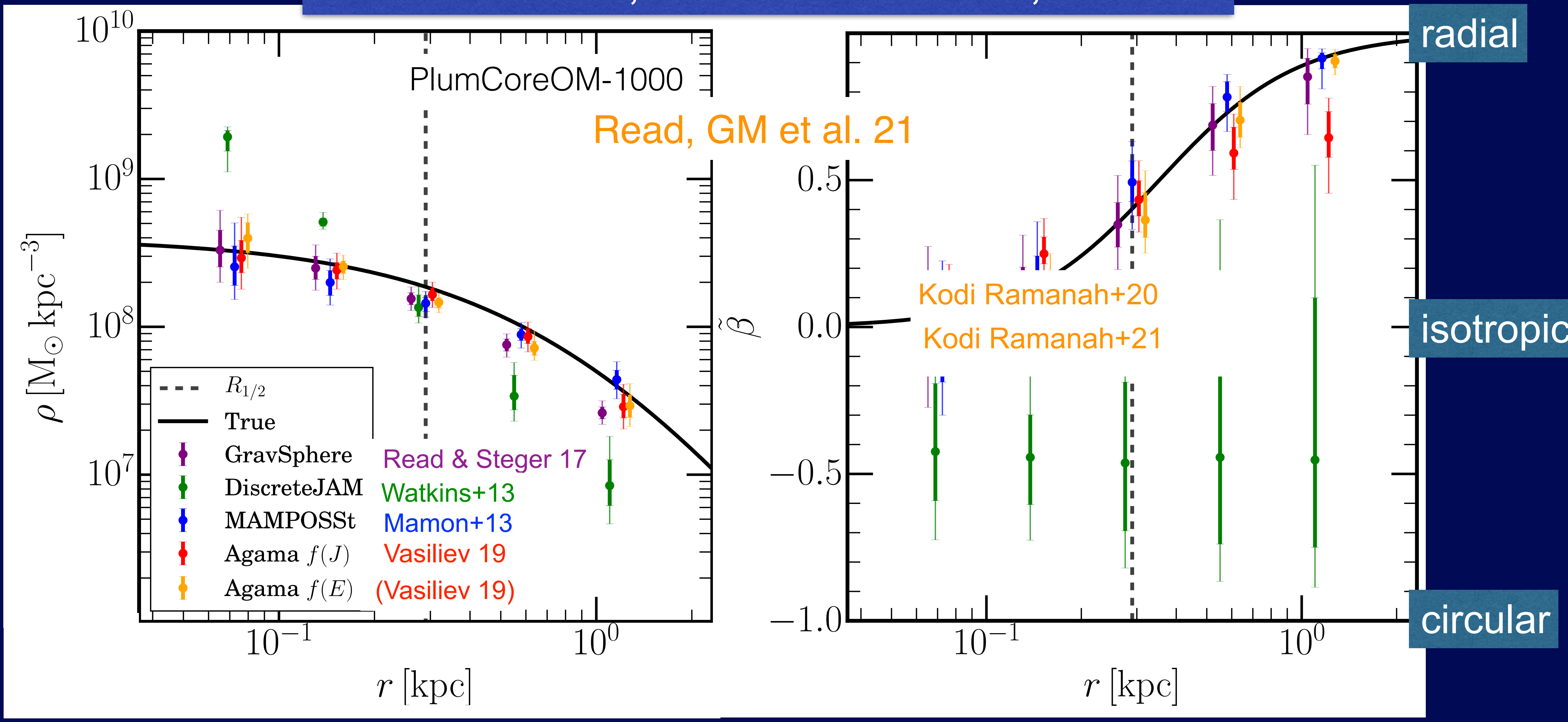


very fast!

Bayesian w MCMC

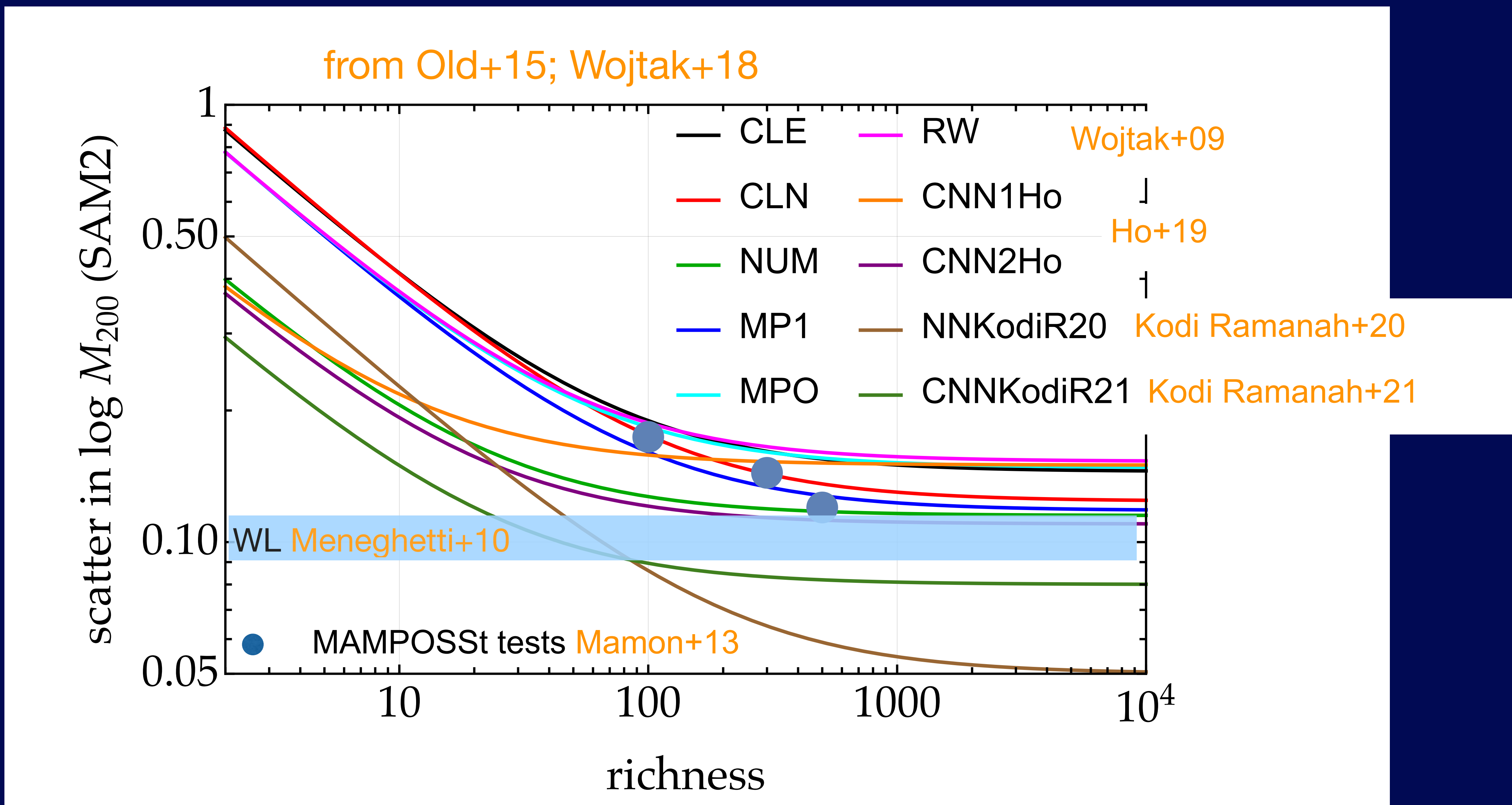
Mass & orbital anisotropy profile challenge: Line-of-sight only

Plummer stars, cored dark matter, *N*=1000



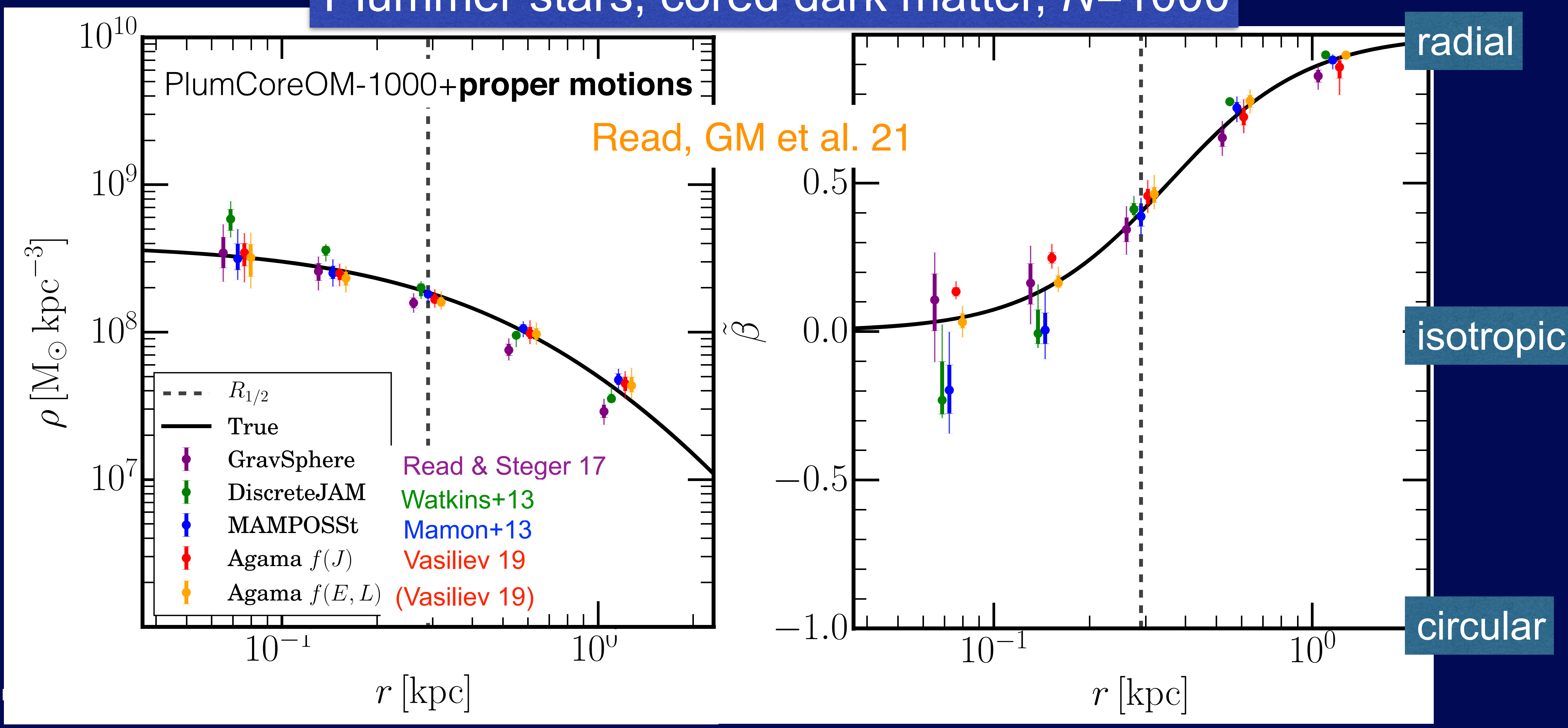
MAMPOSSt competitive on radial profiles of both mass & velocity anisotropy

Comparison with Machine Learning



Mass & orbital anisotropy profile challenge: Line-of-sight + Proper

Plummer stars, cored dark matter, *N*=1000



MAMPOSSt competitive on radial profiles of both mass & velocity anisotropy

Cluster sample

Stacks of 54 regular WINGS clusters

Cava, Biviano, GM+19

$$\langle z \rangle = 0.05$$

$$\left\langle \log\left(\frac{M}{\mathrm{M}_\odot}\right)\right\rangle = 14.8$$

stack by virial radius

3 ways to estimate virial radius:

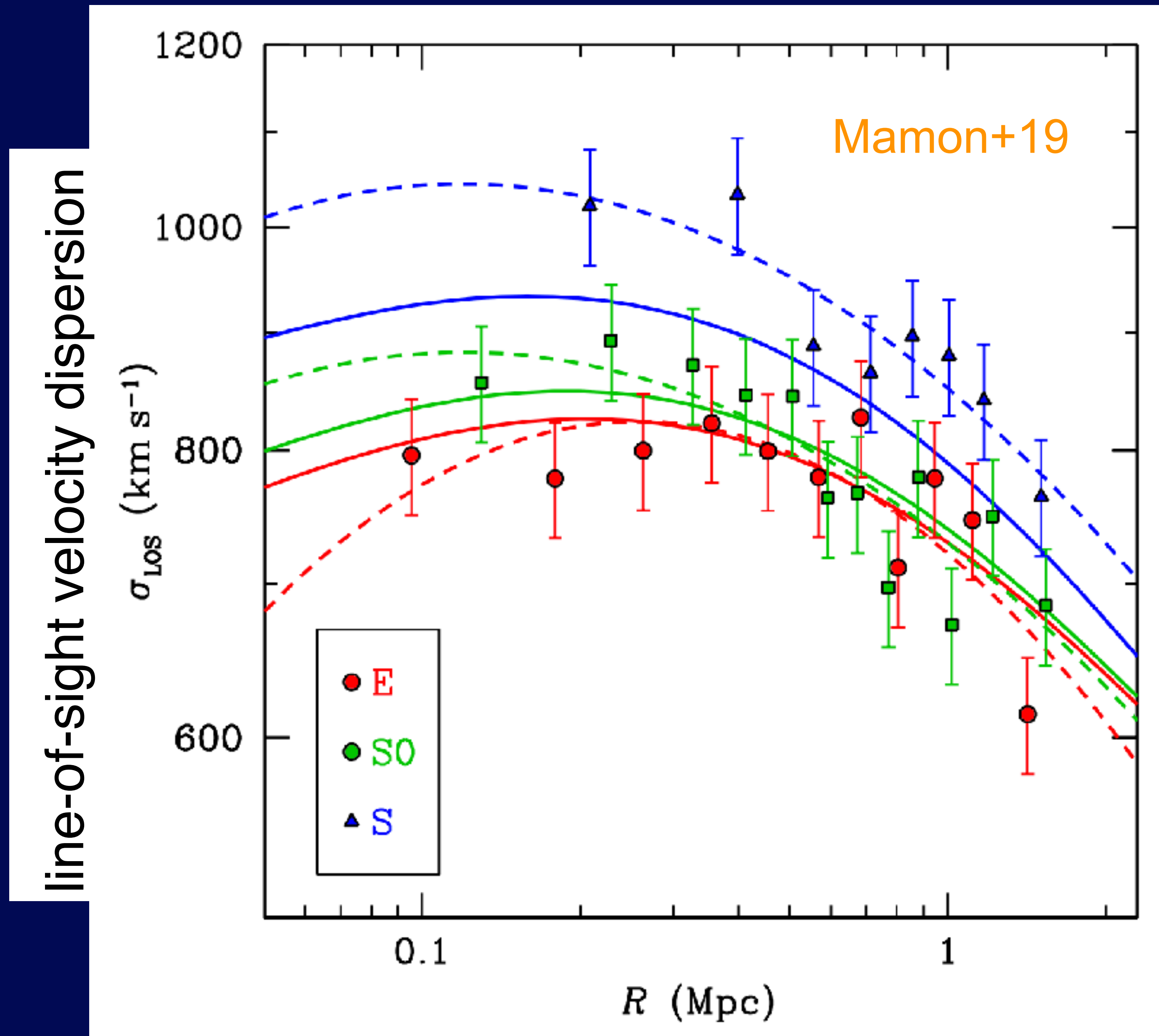
1) velocity dispersion “`sigv`”

2) richness “`Num`"

3) X-ray temperature “`tempX`”

4700 galaxies ($R < r_{\rm vir}$): 1600 E, 1850 S0, 1200 S

Kinematics by morphological class



solid: isotropic, number follows mass
dashed: fit

Statistical analysis method

Bayesian analysis

Likelihood $-\ln \mathscr{L} = -\sum_i \ln p(v_{\rm LOS,i} \,|\, R_i)$

Posterior MCMC (CosmoMCMC)
6 chains of 10 000 $N_{\rm free}$ in //
⟺ ~ 1 million chain elements / model

Priors flat on all log masses and radii, inner slope
Gaussian on tracer surface density profile
(from previous fit on photometric data w cst field)

Model selection: Bayesian evidence

Akaike AICc: $-2 \ln \mathscr{L} + 2\, N_{\rm free}$
BIC: $-2 \ln \mathscr{L} + \ln N_{\rm data}\ N_{\rm free}$

Not 1, not 2

model	mass model		anis.	inner anisotropy			outer anisotropy			TAND	R^{-1}	$-\ln \mathcal{L}_{\rm MLE}$	#	AIC	BIC
	cluster	BCG	model	E	S0	S	E	S0	S				free		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	posterior	(14)	Bayesian evidence	
1	gNFW	–	T	F	F	F	F	F	F	Y	0.004	**33526.28**	12	67076.62	67154.32
2	NFW	NFW	T	F	F	F	F	F	F	N	*0.065*	33526.47	16	67085.05	67188.62

Not 1, not 2, but 30 sets of priors!

model	mass model		anis.	inner anisotropy			outer anisotropy			TAND	R^{-1}	$-\ln \mathcal{L}_{\rm MLE}$	#	AIC	BIC
	cluster	BCG	model	E	S0	S	E	S0	S			posterior	free	Bayesian evidence	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)		(14)		
1	gNFW	–	T	F	F	F	F	F	F	Y	0.004	**33526.28**	12	67076.62	67154.32
2	NFW	NFW	T	F	F	F	F	F	F	N	*0.065*	33526.47	16	67085.05	67188.62
3	NFW	NFW	T	F	F	F	F	F	F	Y	0.012	33526.50	13	67079.08	67163.24
4	gNFW	–	T	F	0	0	F	F	F	Y	0.005	33526.50	10	*67073.05*	67137.80
5	NFW	NFW	gOM	F	F	F	F	F	F	Y	0.007	33526.55	13	67079.18	67163.34
6	gNFW	–	gOM	F	F	F	F	F	F	Y	0.011	33526.79	12	67077.65	67155.34
7	gNFW	–	T	F	F	F	F	F	F	N	*0.040*	33526.91	15	67083.92	67181.01
8	NFW	PS4	T	F	F	F	F	F	F	Y	0.005	33528.33	13	67082.74	67166.90
9	NFW	–	T	F	F	F	F	F	F	Y	0.002	*33528.36*	11	*67078.77*	67150.00
10	NFW	–	T	F	0	0	F	F	F	Y	0.001	33528.41	9	67074.86	67133.14
11	gNFW	–	T	0	0	0	F	F	F	Y	0.005	33528.41	9	67074.86	67133.14
12	NFW	–	T	F	F	F	F	F	F	N	*0.031*	33528.50	14	67085.09	67175.72
13	NFW	–	T	0	0	0	F	F	F	Y	0.002	33528.54	8	67073.11	67124.92
14	NFW	PS4	T	F	F	F	F	F	F	N	*0.043*	33528.55	16	67089.21	67192.77
15	NFW	–	gOM	F	F	F	F	F	F	Y	0.003	33528.92	11	67079.90	67151.12
16	NFW	–	T	0	0	0	0	F	F	Y	0.003	33529.20	7	**67072.42**	67117.76
Einasto (free n)		–	T	0	0	0	0	0	F	Y	0.002	33529.74	6	67073.50	67118.84
18	gNFW	–	T	0	0	0	0	0	F	N	0.015	33529.90	8	67075.83	67127.64
19	gNFW	–	T	0	0	F	0	0	F	Y	0.003	33530.03	8	67076.09	67127.90
20	NFW	–	T	0	0	0	0	0	F	N	0.007	33530.23	7	67074.48	67119.82
21	gNFW	–	T	0	0	0	0	0	F	Y	0.002	33530.27	7	67074.56	*67119.90*
22	NFW	–	T	0	0	F	0	0	F	Y	0.002	33530.35	7	67074.72	67120.06
Einasto (n=6)		–	T	0	0	0	0	0	F	Y	0.002	33530.50	6	67073.02	**67111.88**
24	NFW	–	T	0	0	0	0	0	F	Y	0.001	33530.68	6	67073.38	*67112.24*
25	cNFW	–	T	0	0	0	0	0	F	Y	0.003	33532.30	6	67076.62	67115.48
Hernquist		–	T	0	0	0	0	0	F	Y	0.001	33534.44	6	67080.90	67119.76
27	NFW	–	gOM	0	0	0	0	0	F	Y	0.002	33537.09	6	67086.20	67125.06
28	NFW	–	iso	0	0	0	0	0	0	—	0.002	33538.52	5	67087.05	67119.44
29	NFW	–	T	0	0	0	0	F	0	Y	0.001	33539.46	6	67090.94	67129.80
30	NFW	–	T	0	0	0	F	0	0	Y	0.002	33539.56	6	67091.14	67130.00

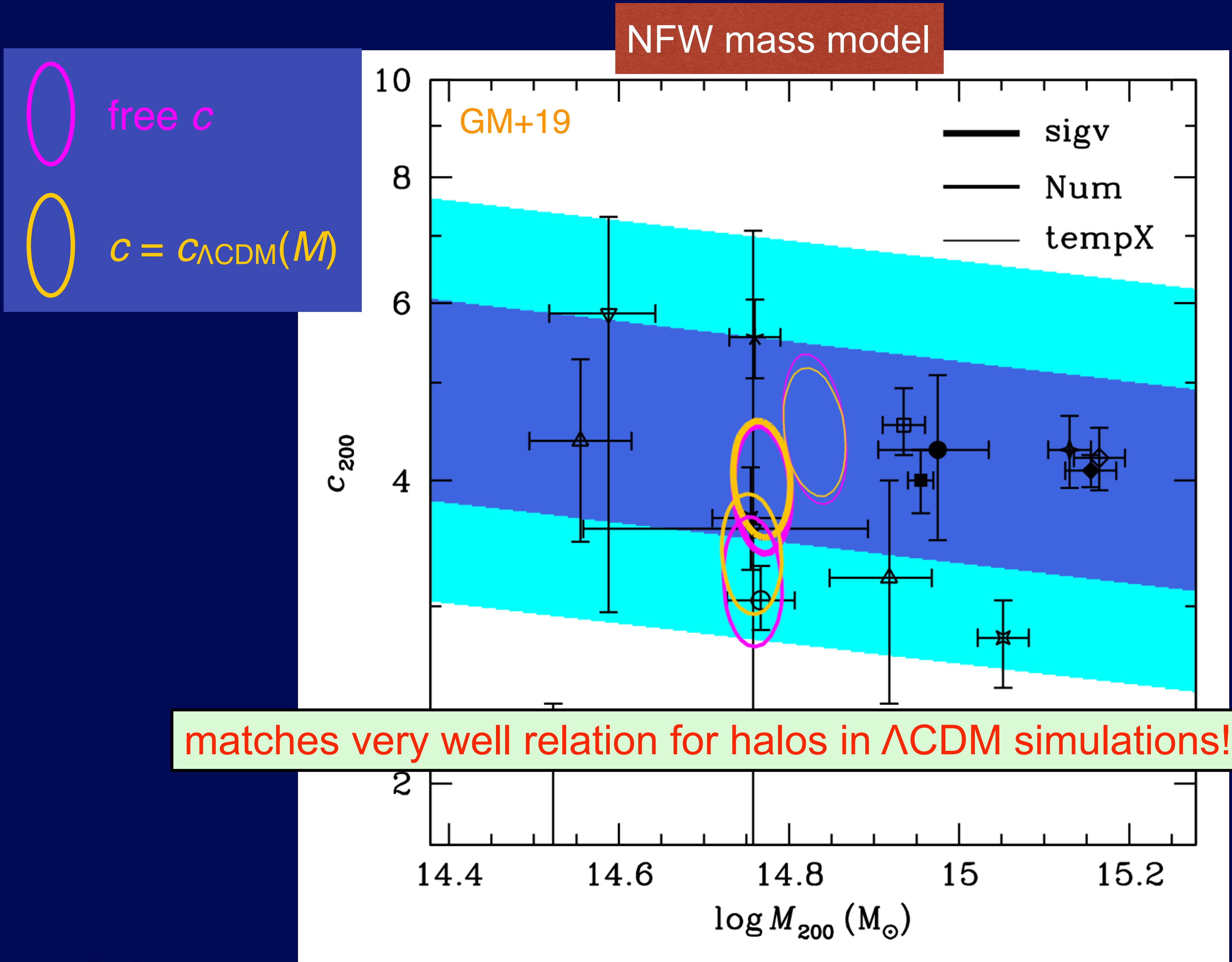
Results

Inner slopes of total mass density profile

Stack	sigv	tempX	Num
inner slope	$-1.4^{+0.5}_{-0.3}$	$-1.7^{+0.3}_{-0.2}$	$-1.8^{+0.4}_{-0.1}$
AICc(gNFW) –AICc(NFW)	1.2	–2.5	–5.9
BIC(gNFW) –BIC(NFW)	>+7	3.7	0.5
	NFW	unclear	~gNFW

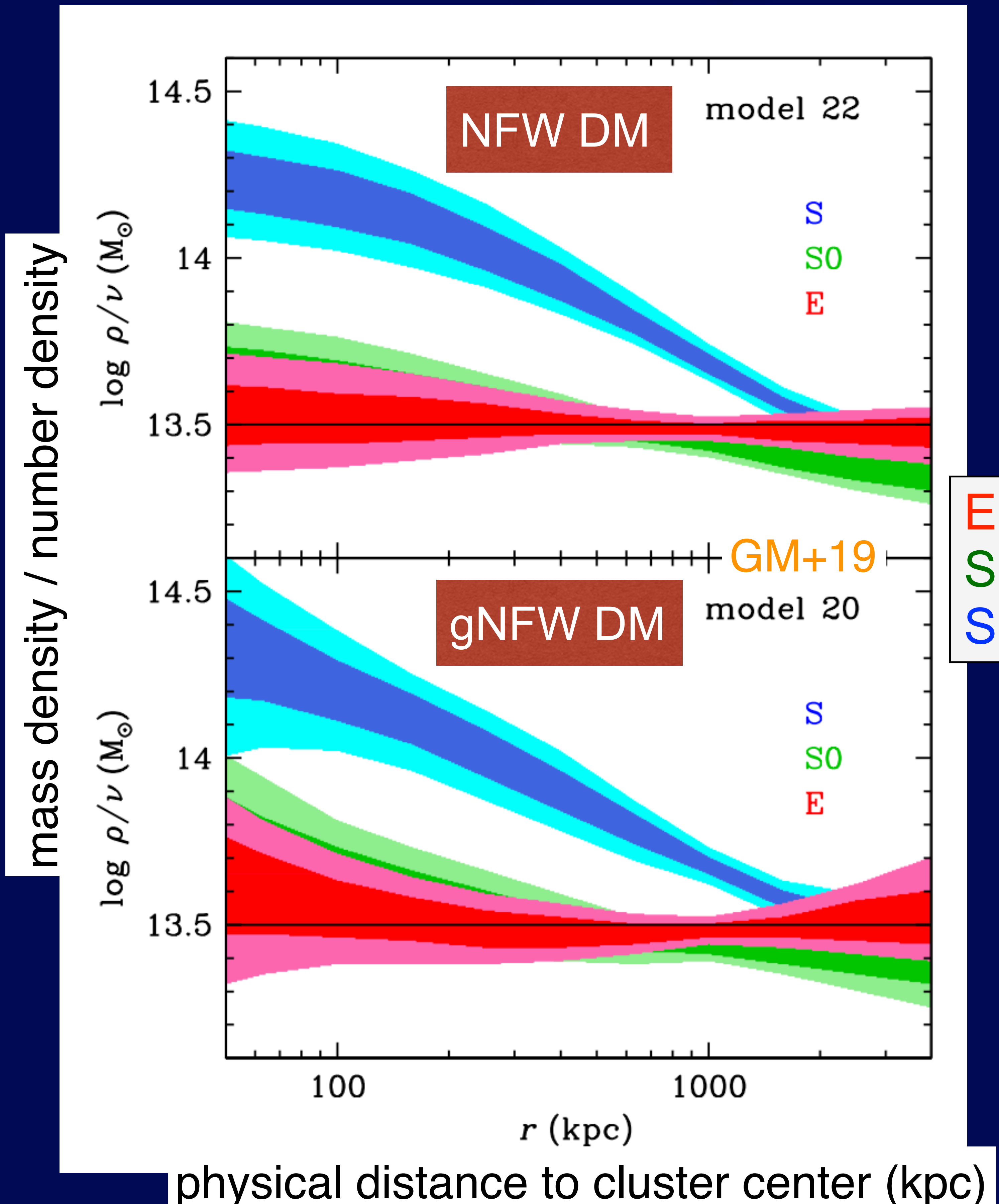
steeper inner slope caused by BCG?

concentration vs. mass



matches very well relation for halos in ΛCDM simulations!

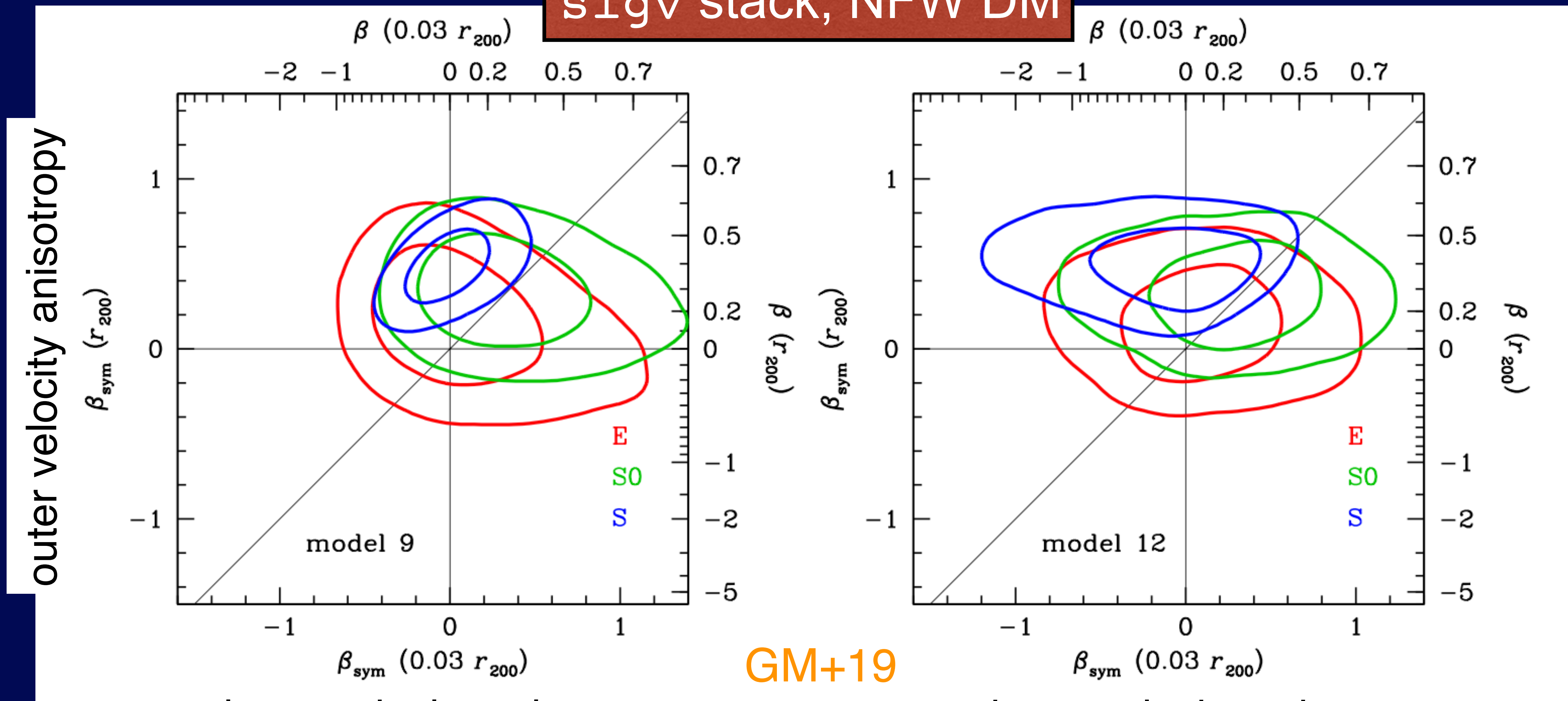
Which morphology traces best the mass?



Ellipticals trace well total mass
S0s fairly well
Spirals trace poorly the total mass

Outer vs. inner Velocity Anisotropy

`sigv` stack, NFW DM



inner velocity anisotropy

inner velocity anisotropy

anisotropy radius = density scale radius

free anisotropy radius

E: isotropic orbits

S: isotropic inner orbits & ~ radial outer orbits

S0: isotropic inner orbits & ~ less radial outer orbits
S0s closer to S than E!

robust to min max radii …
but small changes with stack

Best models

AICc:

NFW,
isotropic inner orbits (all morpho types)
isotropic outer orbits (E)
semi-radial outer orbits (S0, S)

BIC:

n=6 Einasto,
isotropic inner orbits (all morpho types)
isotropic outer orbits (E, S0)
semi-radial outer orbits (S)

Why do ellipticals & S0s have isotropic inner orbits?

morphologically transformed from spirals

2-body relaxation → inefficient!

violent relaxation in merging clusters

1/3 of clusters undergo major mergers since z=1

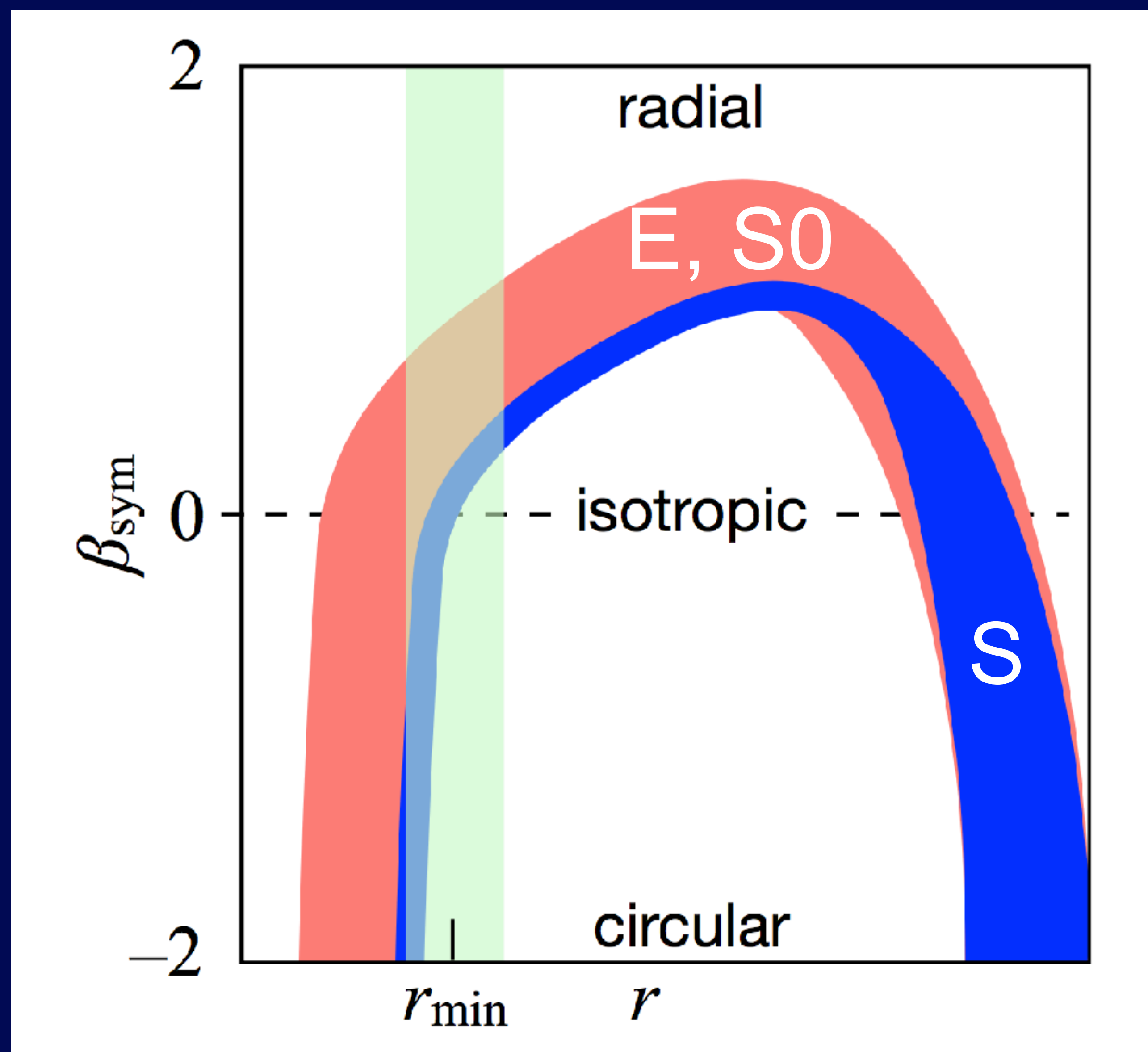
dynamical friction of parent infalling groups

artificial phase mixing of imperfectly stacked halos

Why do spirals have isotropic inner orbits?

4x larger scale radius ⇒ rapid morphological transformation (< 1 orbit)

⇒ narrower range of apocenters & pericenters



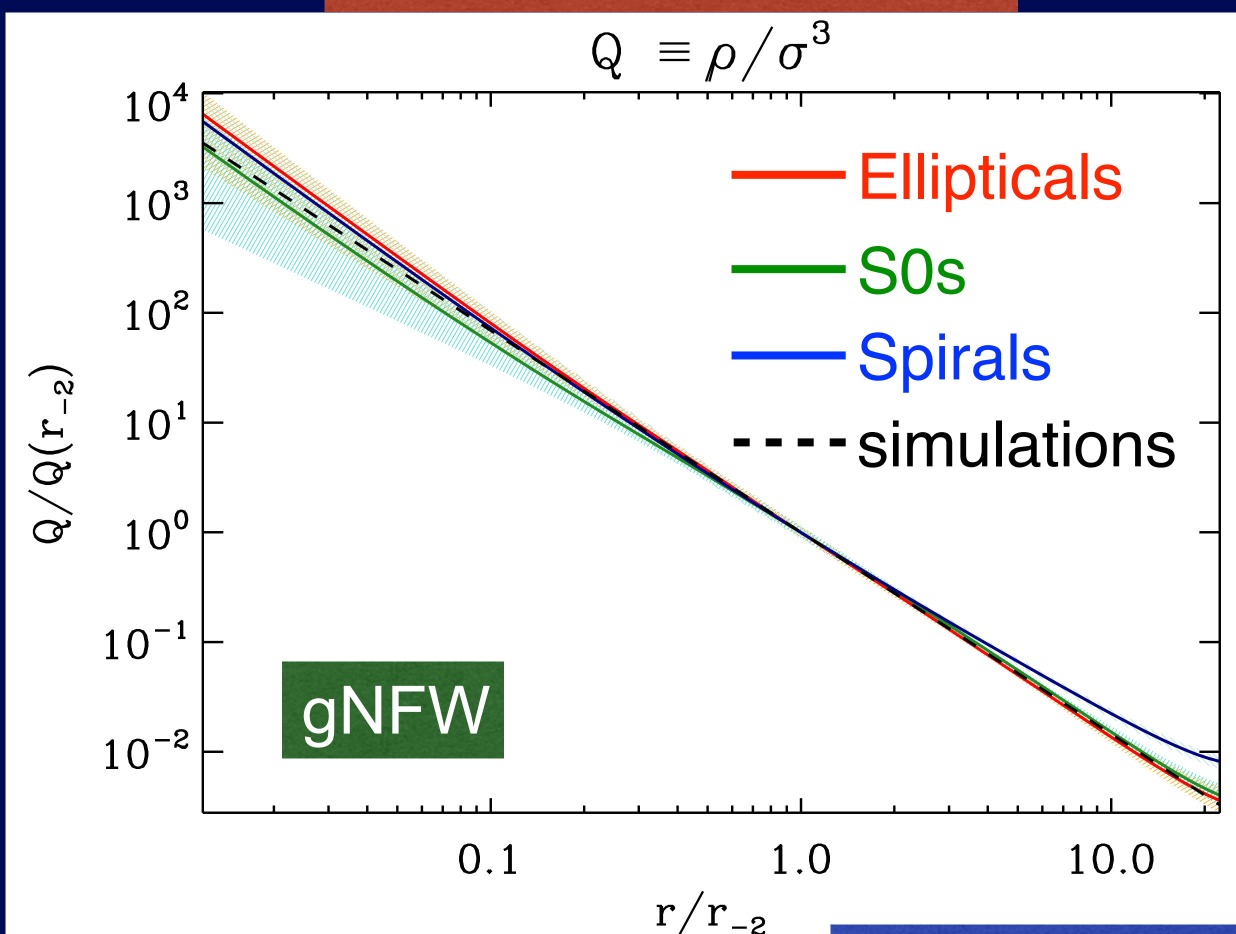
selection effect from rapid morphological transformation of spirals!

Pseudo-phase space density vs. radius

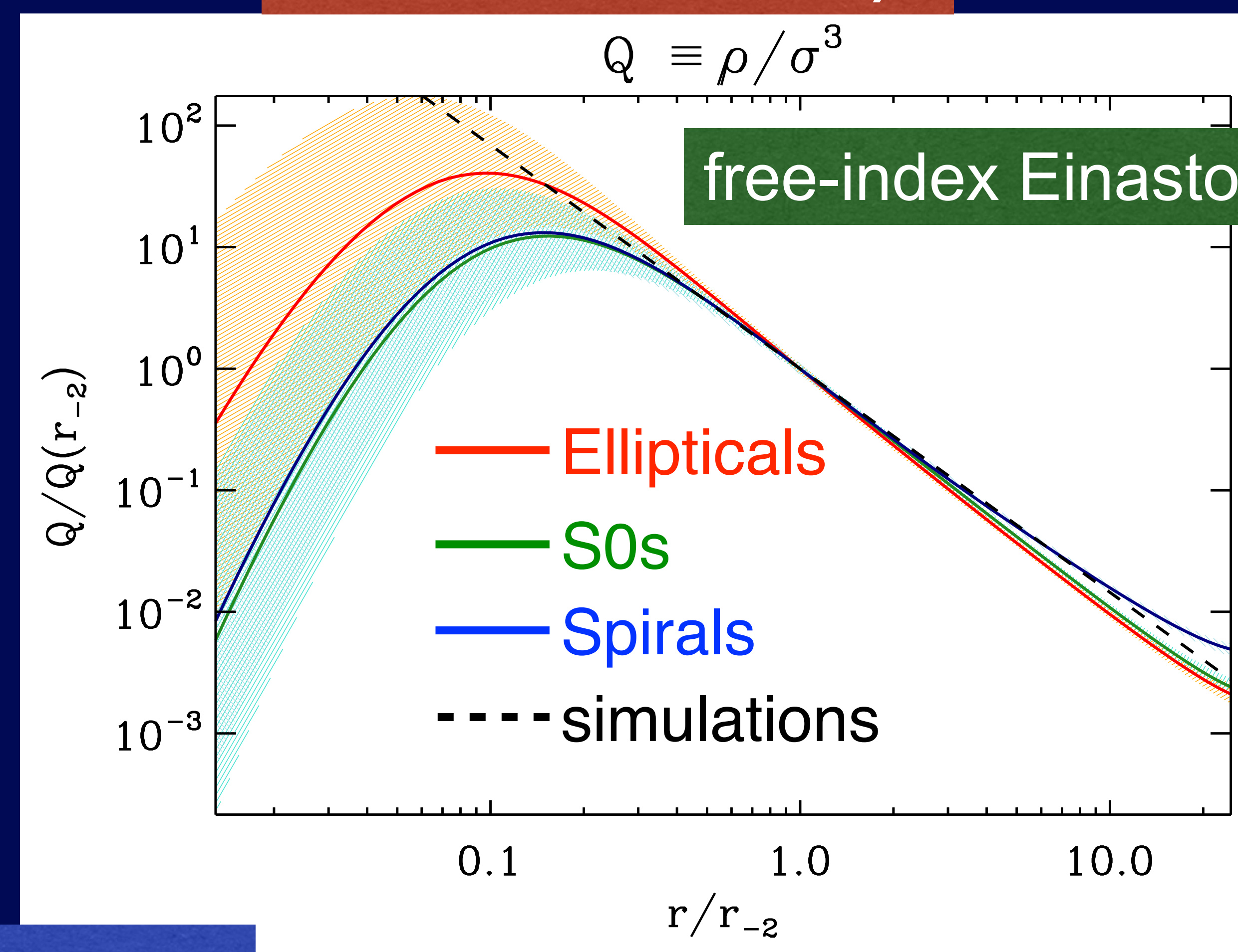
Biviano & GM in prep.

consider all MCMC model parameters (= in proportion to MAMPOSSt posteriors)

total mass density ρ



total mass density ρ

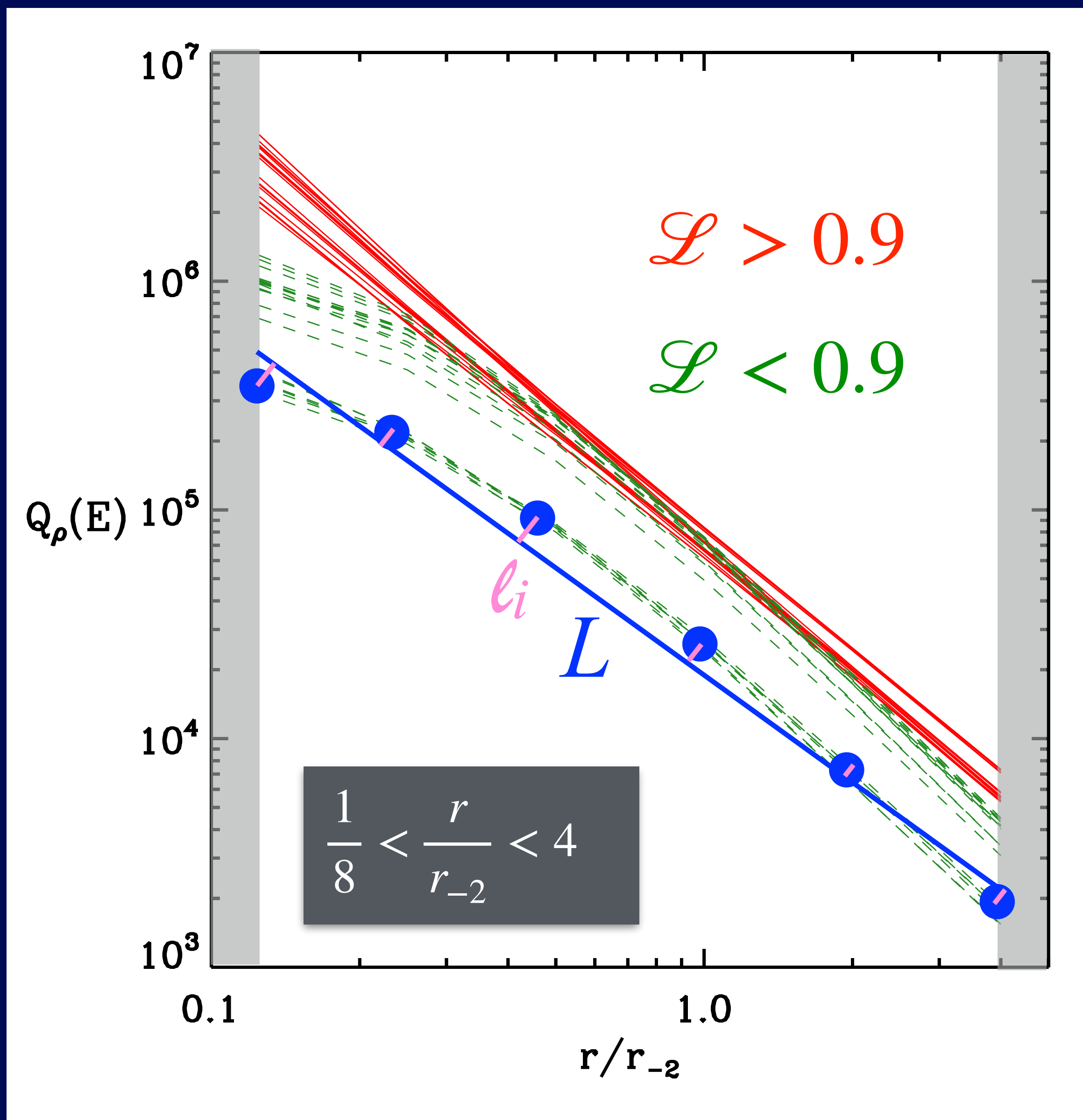


`sigv` stacked cluster

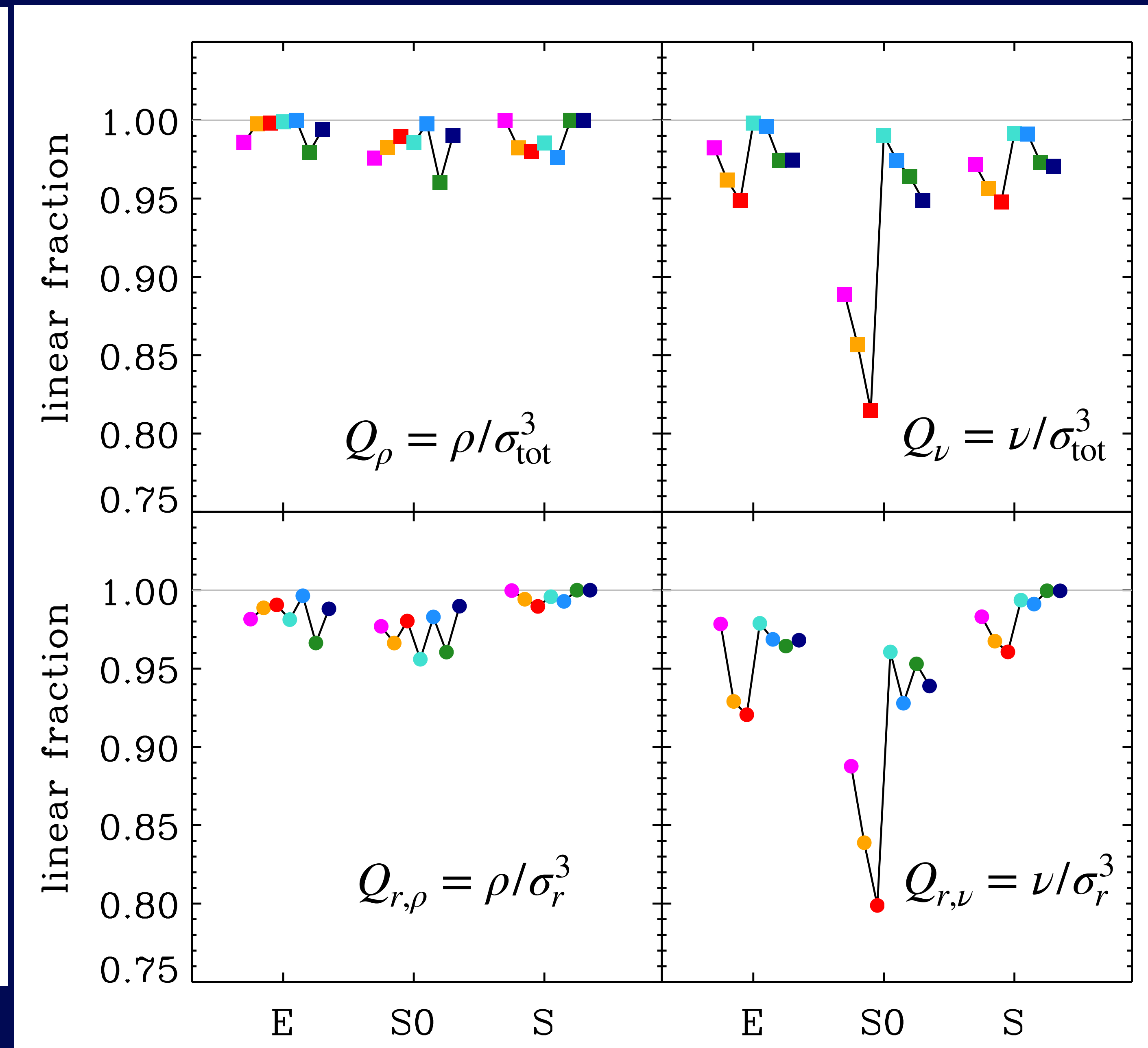
Are scaling relations indeed linear?

Biviano & GM in prep.

colors → ≠ good-fitting mass & anisotropy models



linear: $\mathscr{L} = 1 - \langle\, |\ell_i|\, \rangle / L > 0.9$

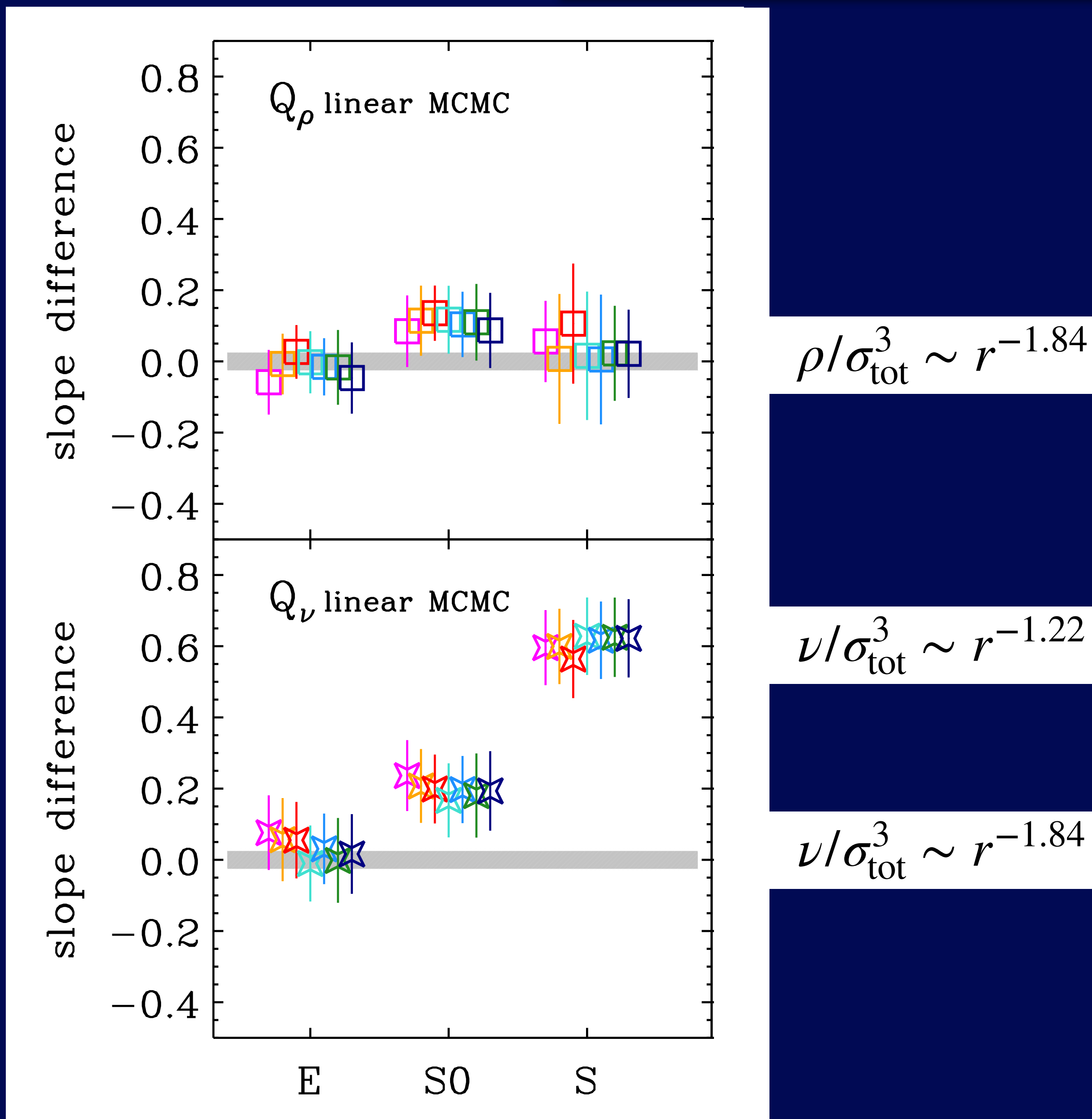


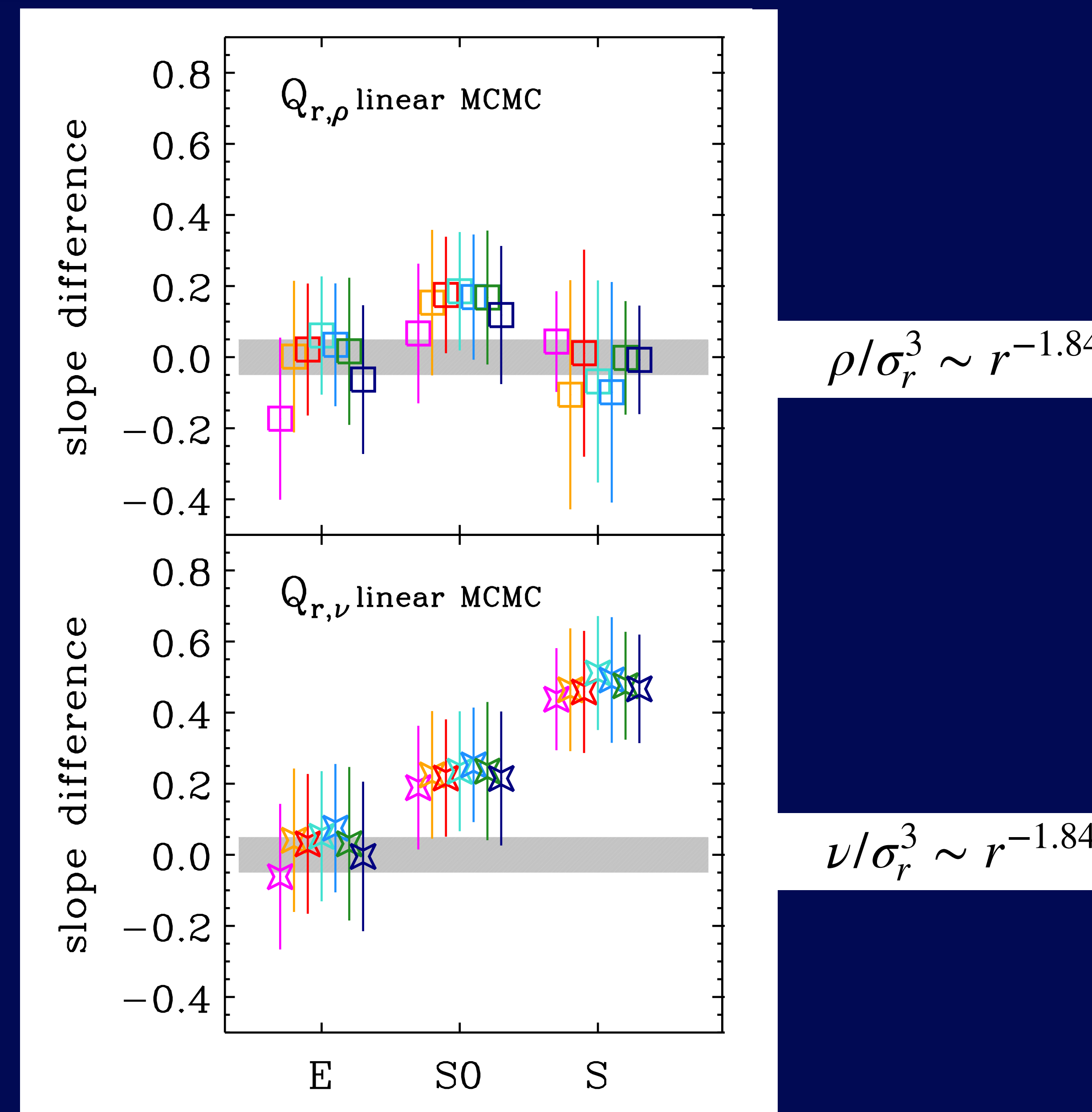
mostly linear relations:
esp. for total mass density (E&S)
worst for number density (S0)

Do linear models have expected slopes?

Biviano & GM in prep.

colors = ≠ best-fitting mass & anisotropy models





Q(*r*) slope:

- as expected for mass density (Es for all models) & number density (Es only)
- = *f* (morph. type & model) for number density

What do we learn from PPSDs?

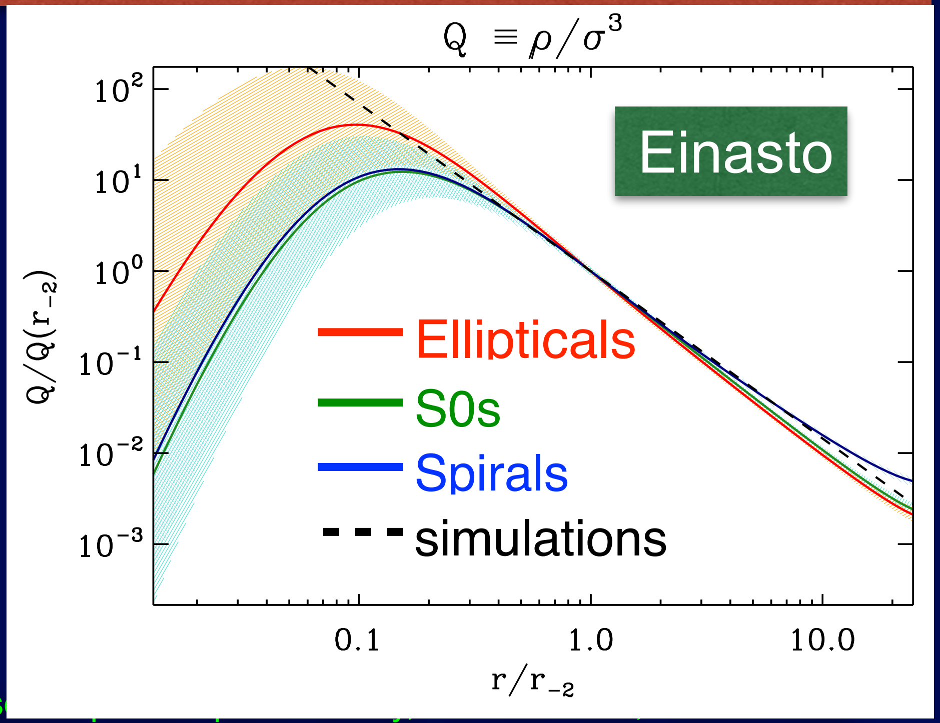
Higher linear fraction for Q_ρ than for Q_ν

+ Slopes of Q_ρ as in dissipationless simulations

$\rightarrow$ PPSD is related to gravitational potential

$\rightarrow$ PPSD $\leftrightarrow$ violent relaxation Colombi 21

or are power-law PPSDs just a coincidence?
(do they extrapolate to low and high radial distances?)



Conclusions

Cluster total mass density profiles	NFW/Einasto or possibly steeper (BCG?)
concentration vs. mass	consistent with cosmological simulations
Galaxies vs. mass	Ellipticals trace mass best, spirals poorly S0s closer to ellipticals
Outer orbits in clusters	S more radial, E more isotropic, S0s in between, closer to S
Inner orbits in clusters	isotropic! E/S0: violent relax'n & dyn'l friction of groups S: selection effect of small range of pericenters
pseudo-phase space density	driven by gravitational potential?