Determining ground layer turbulence statistics using a SLODAR-type method

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Joint work with:
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Introduction

- SLODAR-type methods aim to recover the vertical turbulence profile from the cross-correlation of wavefront slope measurements from two guide stars
- The turbulence profile is crucial for AO systems with multiple guidestars, where strong prior information is required to stabilize the tomography problem
- SLODAR methods rely on the Kolmogorov/von Kármán models for turbulence statistics
- However, turbulence statistics at the ground can deviate from this model
- This issue is emphasised by the fact that often much of the turbulence strength is located close to the ground
- What if we could use the same measurements to both reconstruct the turbulence profile and infer a model for the ground layer turbulence simultaneously?

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- SLODAR is a well-posed problem (fewer unknowns than measurements)
- Using inverse problems methods, we can extract more information from the same measurements
- The goal of our method is to simultaneously reconstruct the vertical turbulence profile of the atmosphere and the power spectral density at the ground layer
- This could be particularly valuable in poor imaging conditions, where better prior information about the turbulence is needed to offset increased measurement errors

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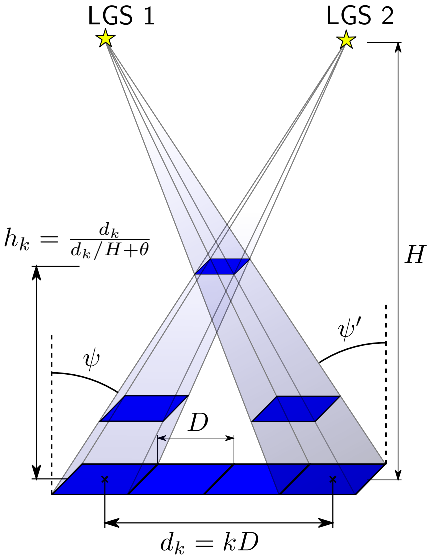
Outline of this talk

1. Brief description of SLODAR
2. Solving for PSD with SLODAR
3. Numerical results
4. Open questions

Crash course to SLODAR

- Measurement setup with two LGS wavefront sensors
- Correlation of measurements from two subapertures (one from each WFS) gives information about turbulence at an altitude depending on the distance between the subapertures
- SLODAR-type methods aim to reconstruct the vertical turbulence profile from these correlations
- Layer altitudes depend on LGS altitude H, LGS separation θ, and subaperture distances d_k



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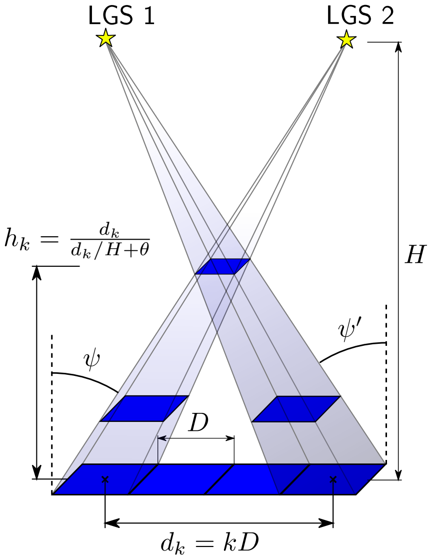

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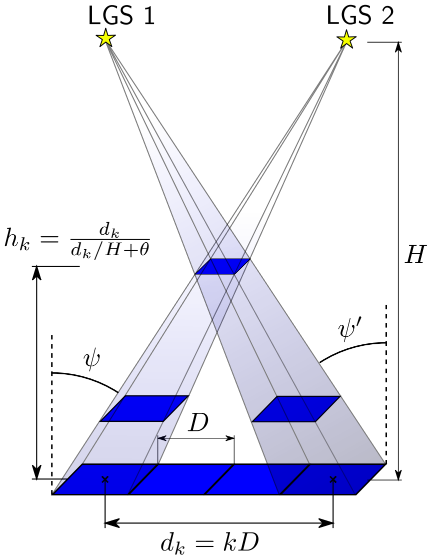

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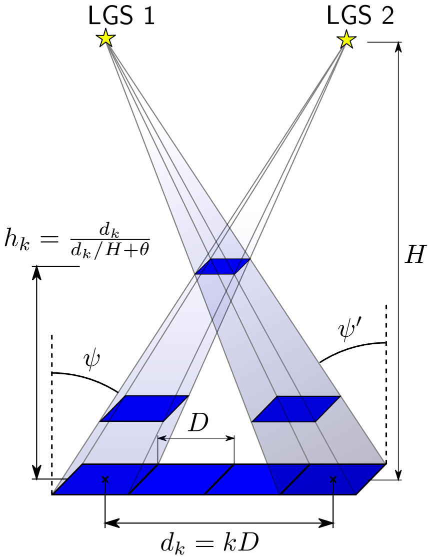

Crash course to SLODAR

- Correlation of x-directional slope derivatives for subaperture separation $\boldsymbol{d}_j$:

$$\mathrm{Cor}^x(\boldsymbol{d}_j) = \sum_k \frac{1}{D^4} \left(\int_{\mathbb{R}^2} e^{-2\pi i \boldsymbol{\xi} \cdot (\eta_k \boldsymbol{d}_j - h_k \boldsymbol{\theta})} \Psi_0(\boldsymbol{\xi}) |g_k^x(\boldsymbol{\xi})|^2 d\boldsymbol{\xi} \right) \rho_k$$

  - ρ_k: turbulence strength at layer k
  - $\Psi_0(\boldsymbol{\xi})$: von Kármán power spectral density, $\Psi_0(\boldsymbol{\xi}) = 0.0229(|\boldsymbol{\xi}|^2 + L_0^{-2})^{-11/6}$
- Collecting the integrals into a matrix $\boldsymbol{A}^x$ (and similarly for y-slopes), we get

$$\boldsymbol{b} := \begin{pmatrix} \boldsymbol{b}^x \\ \boldsymbol{b}^y \end{pmatrix} = \begin{pmatrix} \boldsymbol{A}^x \\ \boldsymbol{A}^y \end{pmatrix} \boldsymbol{\rho} =: \boldsymbol{A}\boldsymbol{\rho}$$

  - $\boldsymbol{\rho}$: vector of turbulence strengths ρ_k at altitudes h_k
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- Easy to solve in the least squares sense (e.g. using Matlab's quadprog):

$$\min_{\boldsymbol{\rho} \geq 0} \|\boldsymbol{A}\boldsymbol{\rho} - \boldsymbol{b}\|_2^2$$

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Modified SLODAR method

- Simple idea: for the ground layer $h_0 = 0$, replace Ψ_0 by an unknown PSD Ψ:

$$\rho_0 \Psi(\xi) = \sum_{l=0}^{N_r - 1} \psi_l f_l(|\xi|)$$

- Note: ρ_0 included in above definition to avoid nonlinearity
- It follows that

$$A_{j0}^x \rho_0 = \frac{1}{D^4} \sum_{l=0}^{N_R - 1} \left(\int_{\mathbb{R}^2} e^{-2\pi i \boldsymbol{\xi} \cdot \eta_0 \boldsymbol{d}_j} |g_k^x(\boldsymbol{\xi})|^2 f_l(|\boldsymbol{\xi}|) d\boldsymbol{\xi} \right) \psi_l$$

- As before, collect the integrals into a matrix $\boldsymbol{B}$ to obtain

$$\boldsymbol{b} = \begin{pmatrix} \boldsymbol{b}^x \\ \boldsymbol{b}^y \end{pmatrix} = \begin{pmatrix} \boldsymbol{A}^x \\ \boldsymbol{A}^y \end{pmatrix} \boldsymbol{\rho} + \begin{pmatrix} \boldsymbol{B}^x \\ \boldsymbol{B}^y \end{pmatrix} \boldsymbol{\psi} =: \boldsymbol{A\rho} + \boldsymbol{B\psi}$$

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- Discretizing the PSD leads to many more unknowns than measurements
- The system $\boldsymbol{b} = \boldsymbol{A\rho} + \boldsymbol{B\psi}$ is strongly underdetermined $\Rightarrow$ inverse problem
- Standard methods (e.g. least-squares solution) fail: noise in the measurement b dominates
- Even worse, ψ decays rapidly since it is (close to) a power law
- Solution: regularization!
- Use prior information to favor solutions resembling a power law

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- **Method 1: Parametric Power Law.** Seek solutions of the form

$$\psi_j(c,\gamma) = c(r_j^2 + 1/L_0^2)^{-\gamma}$$

- Least-squares solution:

$$\min_{\boldsymbol{\rho},c,\gamma\geq 0} \|\boldsymbol{A}\boldsymbol{\rho} + \boldsymbol{B}\boldsymbol{\psi}(c,\gamma) - \boldsymbol{b}\|_2^2$$

  - Regularization by discretization
  - Nonlinear, but low number of parameters makes it easy to solve

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- **Method 2: Tikhonov Regularization.** Solve least-squares problem

$$\min_{\boldsymbol{\rho}\geq 0, \boldsymbol{\psi}\geq 0} \|\boldsymbol{A\rho} + \boldsymbol{B\psi} - \boldsymbol{b}\|_2^2 + \beta\|\boldsymbol{\Gamma}(\boldsymbol{\psi} - \boldsymbol{\psi}_0)\|_2^2 + \beta'\|\boldsymbol{\Gamma}'(\boldsymbol{\psi} - \boldsymbol{\psi}_0)\|_2^2$$

  - $\boldsymbol{\Gamma}$, $\boldsymbol{\Gamma}'$ and $\boldsymbol{\psi}_0$ encode our prior information about the unknown PSD $\boldsymbol{\psi}$
  - β and β' control how strongly prior information is enforced
- Prior PSD ψ_0 given by method 1
- Intuition for choosing Γ and Γ': impose a penalty on the relative error between ψ and ψ_0, and similarly between their derivatives
- Easy to solve using quadratic programming
- Slightly more complex than method 1, but much more suitable for cases where true solution is not quite a power law

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Numerical simulations

- Simulations using MOST, an AO system simulation toolkit developed at JKU
- Telescope diameter $42\mathrm{m}$
- WFSs with 84×84 subapertures (i.e. $D = 0.5\mathrm{m}$)
- LGS separation $\boldsymbol{\theta} = (7.5, 0)$, in arcminutes
- $N_L = 61$ layers
- $\boldsymbol{d}_k = (k, 0)$ with $k = 0, \ldots, 60$
- $H = 90\mathrm{km}$
- $h_k = \frac{kD}{kD/H + |\boldsymbol{\theta}|} \Rightarrow h_0 = 0\mathrm{m}$, $h_{60} \approx 12\mathrm{km}$
- Measurements over 50 timesteps
- Ground layer PSD discretized by 400 points r_i, with $r_{\mathsf{max}} = 10$

Numerical simulations

- Simulations using MOST, an AO system simulation toolkit developed at JKU
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- Simulations using MOST, an AO system simulation toolkit developed at JKU
- Telescope diameter $42\mathrm{m}$
- WFSs with 84×84 subapertures (i.e. $D = 0.5\mathrm{m}$)
- LGS separation $\boldsymbol{\theta} = (7.5, 0)$, in arcminutes
- $N_L = 61$ layers
- $\boldsymbol{d}_k = (k, 0)$ with $k = 0, \ldots, 60$
- $H = 90\mathrm{km}$
- $h_k = \frac{kD}{kD/H + |\boldsymbol{\theta}|} \Rightarrow h_0 = 0\mathrm{m}$, $h_{60} \approx 12\mathrm{km}$
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- von Kármán power spectral density given by

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- For ground layer $k = 0$, we changed the PSD to

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PSD reconstruction with method 1

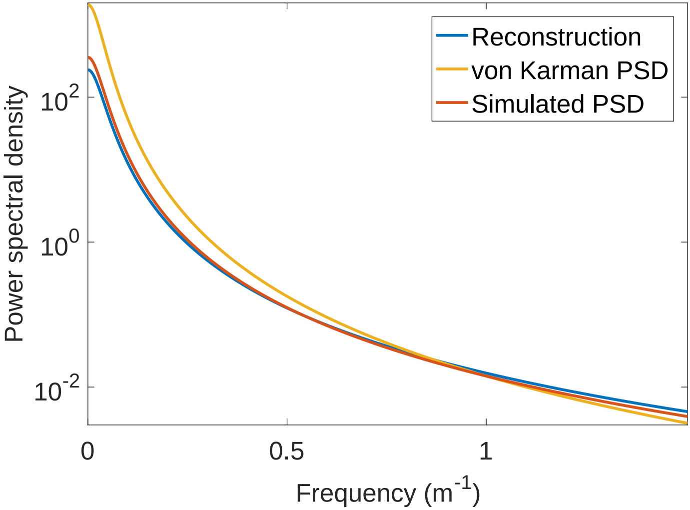
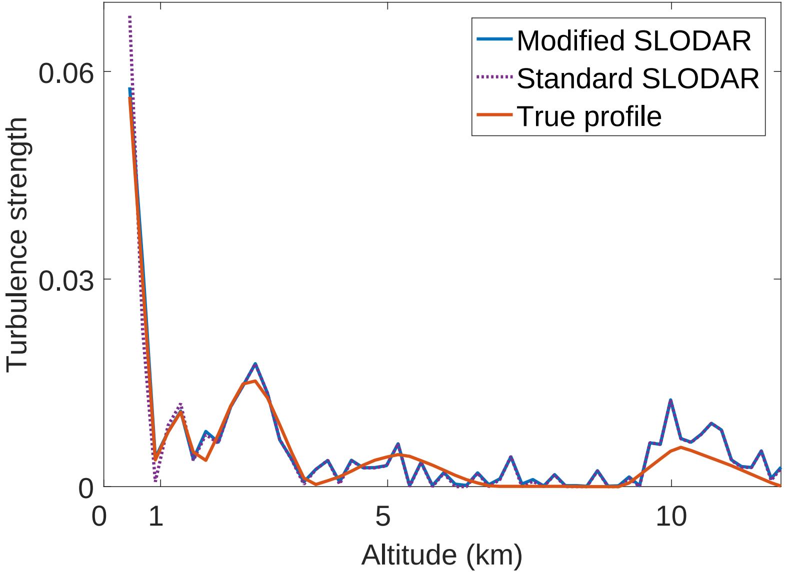
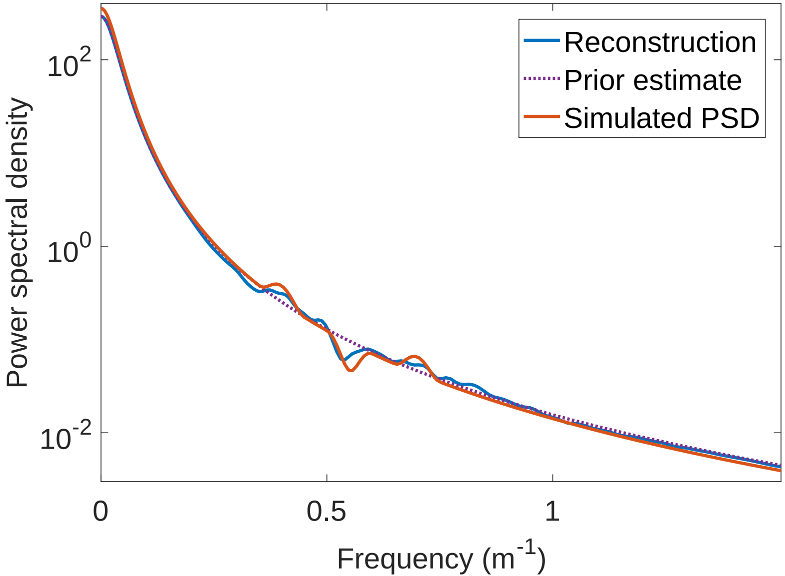


Figure: Simulated PSD exponent -1.5732, reconstructed exponent -1.497.

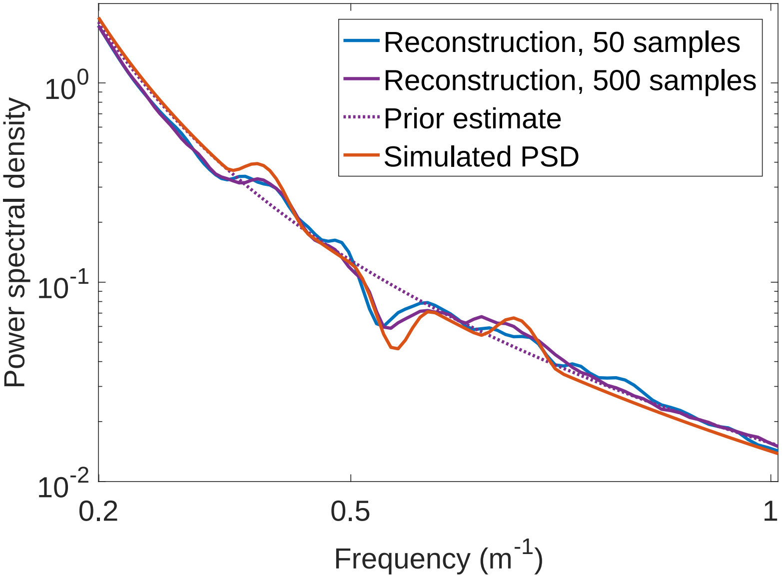
Turbulence profile reconstruction with method 1



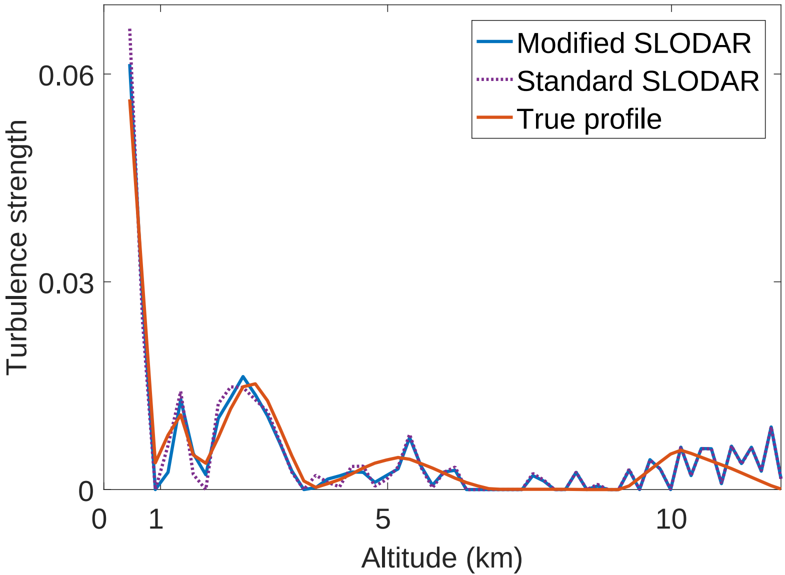
PSD reconstruction with method 2, using 50 timesteps



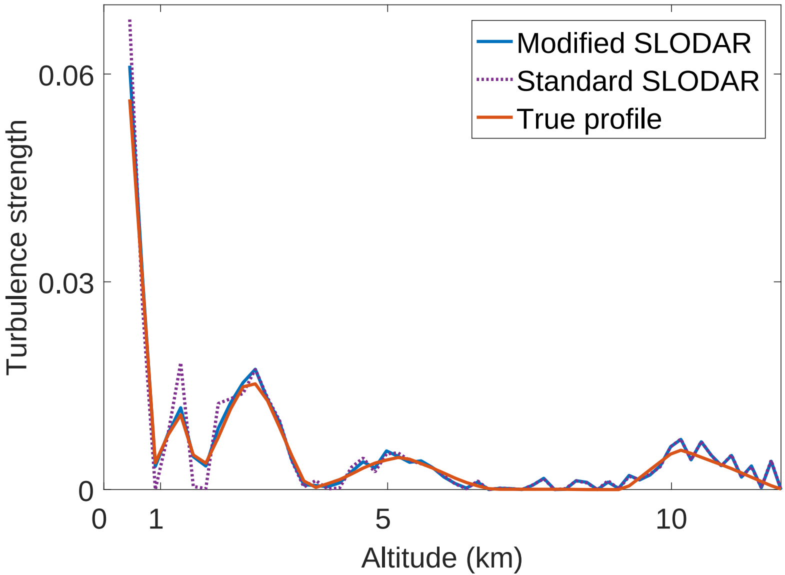
PSD reconstruction with method 2, using 500 timesteps



Turbulence profile, method 2, using 50 timesteps



Turbulence profile, method 2, using 500 timesteps



Open questions

- What is a realistic class of models for the ground layer turbulence statistics?
- How much does real low altitude turbulence deviate from an isotropic and homogeneous model?
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- Could we reliably reconstruct L_0 as well?

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THE END

Thank you for your attention!