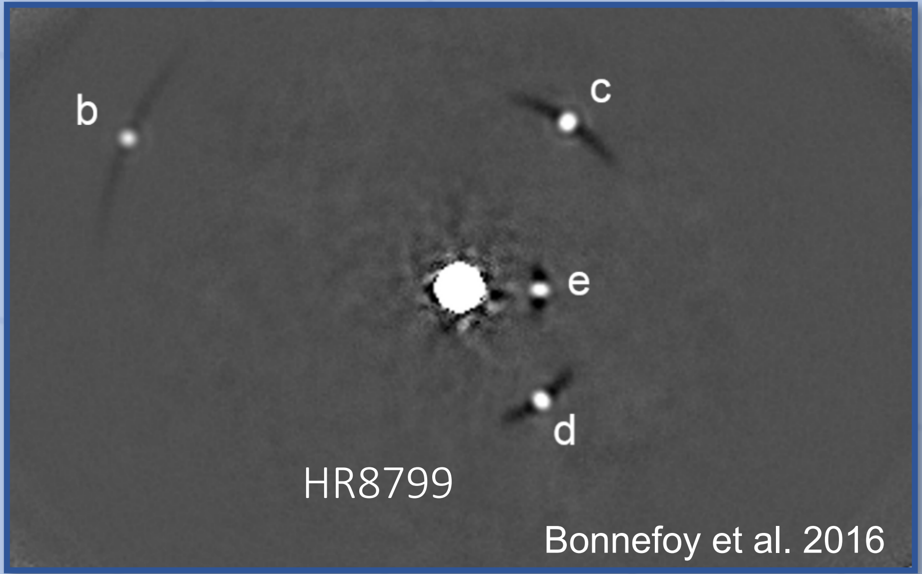
Pair-based Analytical model for Segmented Telescopes Imaging from Space (PASTIS) for sensitivity analysis

L. Leboulleux, J.-F. Sauvage, L. Pueyo, T. Fusco, R. Soummer, A. Sivaramakrishnan, J. Mazoyer, M. N'Diaye, O. Fauvarque

TOWARD EXO-EARTH IMAGING



SPHERE/IRDIS *K*-BAND

	HR 8799 e	Exo-Earth
Contrast ratio	10^{-4}	$< 10^{-10}$
Angular separation with star	$0.368''$	$< 0.1''$

▶ **Two main challenges: Angular separation and contrast between star and planet**

SEGMENTED TELESCOPES

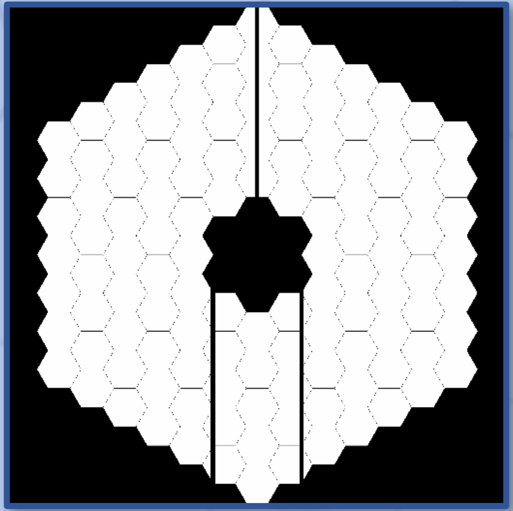
Angular separation → Atmosphere → Space telescopes

Angular separation → Primary mirror size → Large telescopes

Space telescopes / Large telescopes → Weight + shape in space craft → Segmented space telescopes → Cophasing / Stability / Exotic pupil and PSF

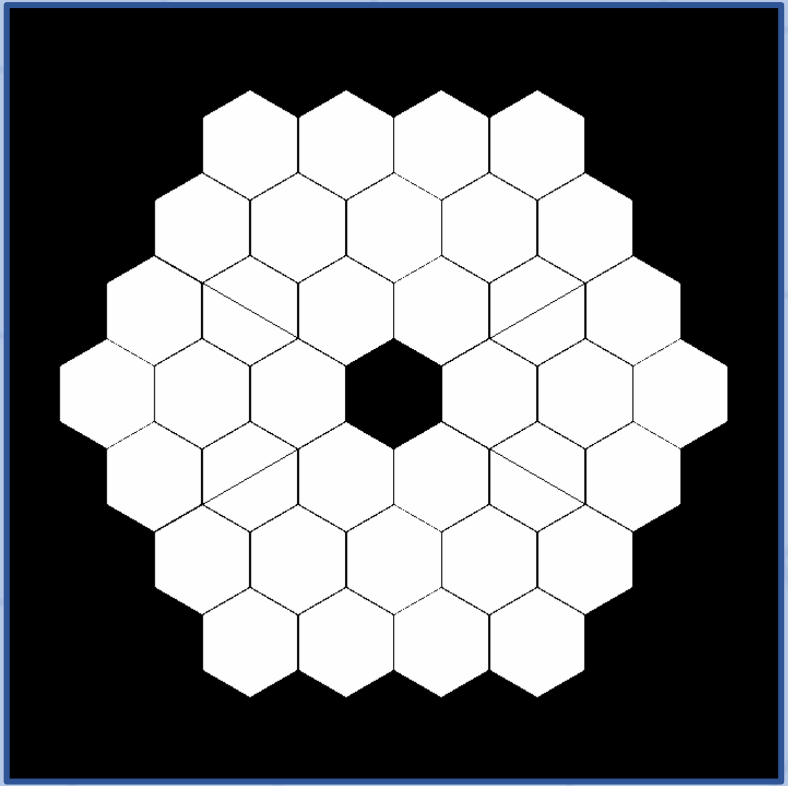
Contrast → Starlight → Coronagraphy

Contrast → Speckles → WFS + WFC (deformable mirrors)



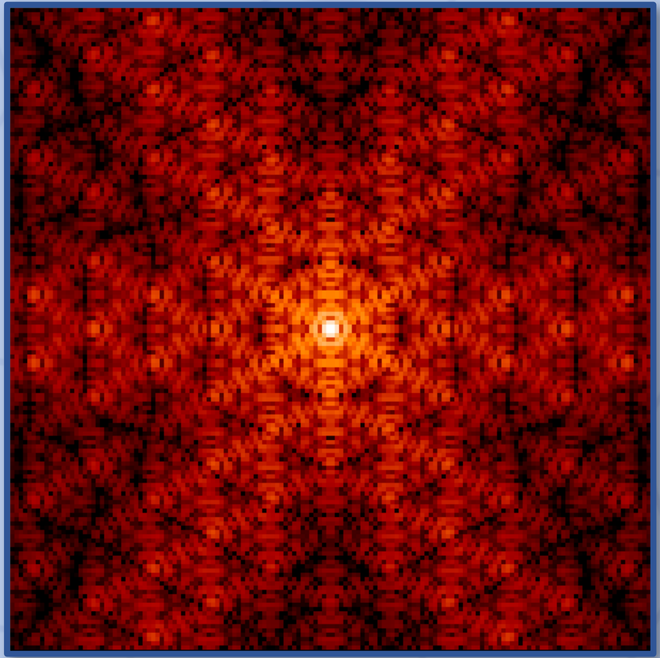
LUVOIR A
primary mirror

LUVOIR-LIKE PUPIL

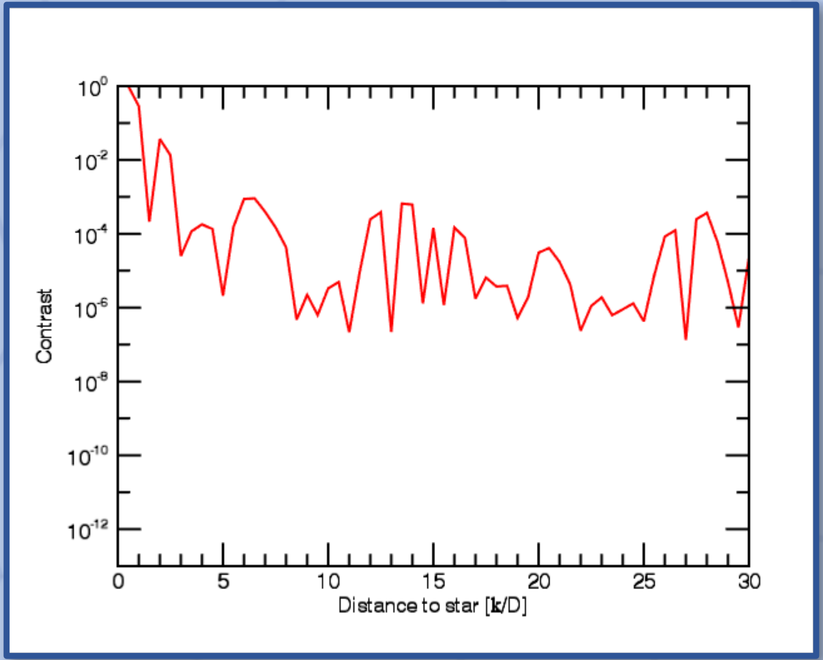


LUVOIR-LIKE PUPIL

WITHOUT CORONAGRAPH

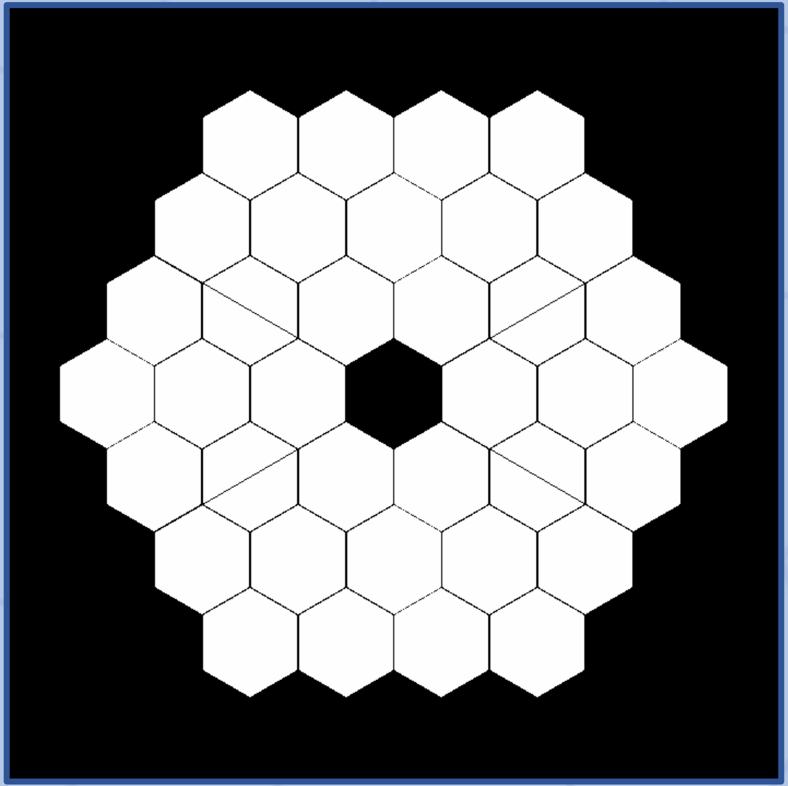


PSF



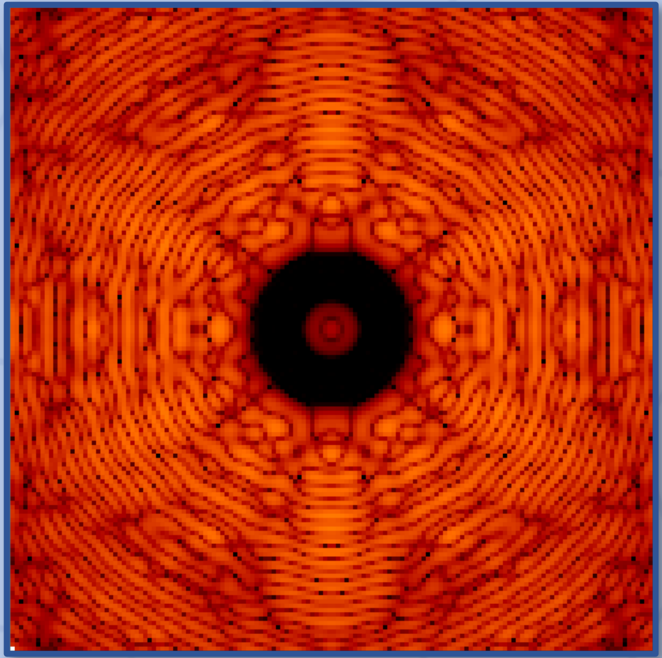
Radial cut of the PSF

LUVOIR-LIKE PUPIL

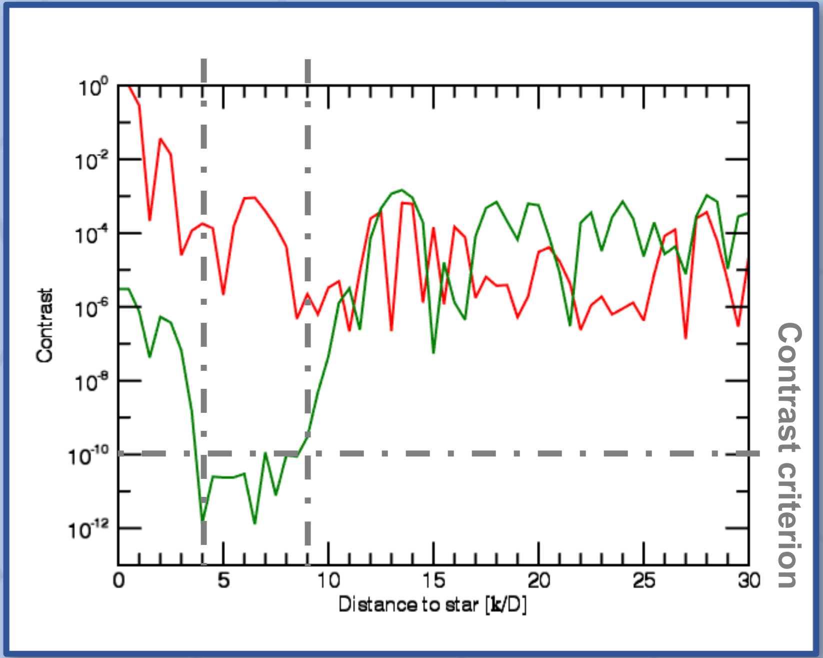


LUVOIR-LIKE PUPIL

WITH CORONAGRAPH (APLC – N'Diaye et al. 2016)



PSF



Radial cut of the PSF

- Reaching the criteria is possible in simulations but how to maintain it?

➡ Need for an error budget

ERROR BUDGET

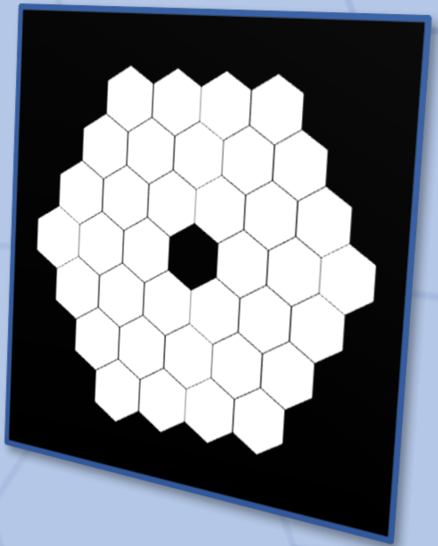
Tolerancing on segmented telescopes for high-contrast performance

Identification of the sensitive factors limiting the performance on segmented telescopes

Quantification of their impact

Cophasing
with piston, tip, and tilt errors

Local higher-order Zernike polynomials

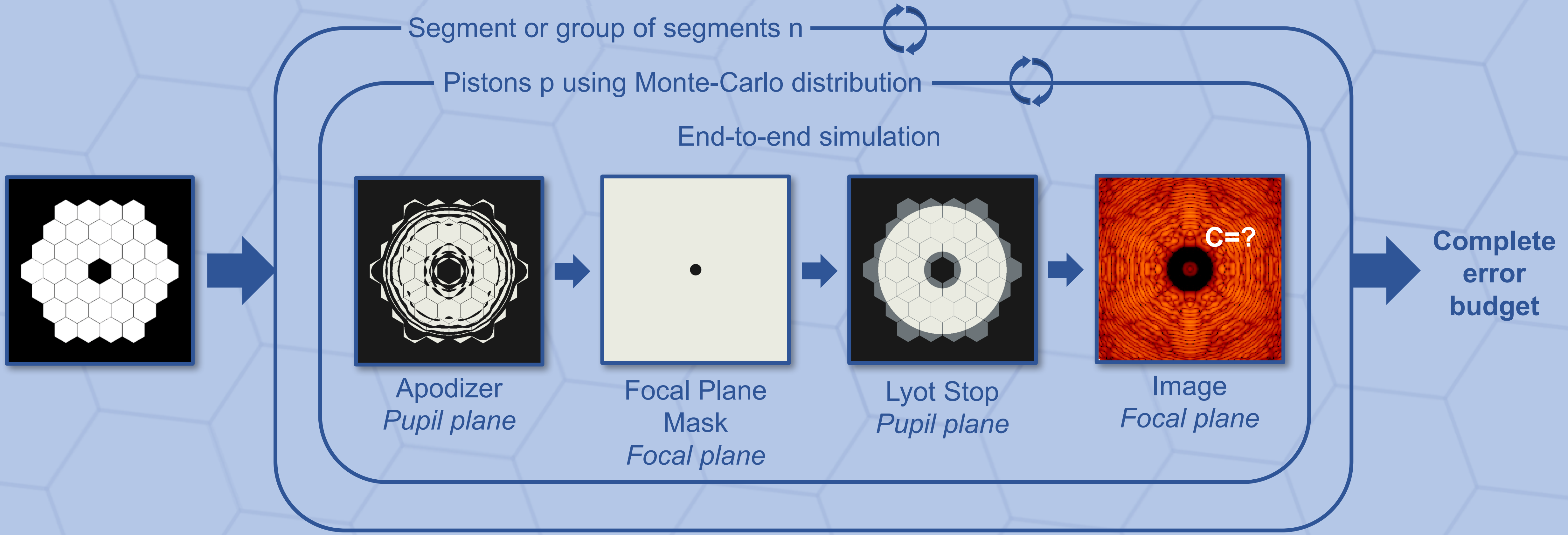


Segment-dependant sensitivity

Vibrations or resonant modes on the segments

TRADITIONAL METHOD

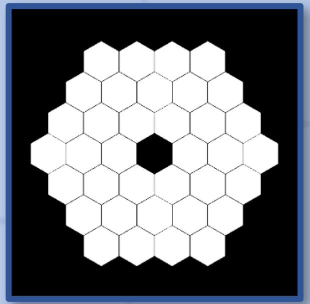
Method based on multiple end-to-end simulations



▶ **Time- and computer-consuming**

NEED FOR AN ANALYTICAL MODEL

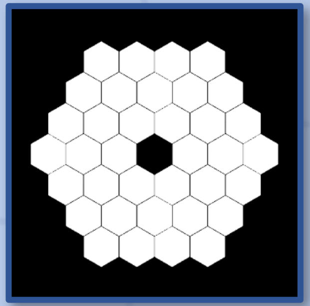
Method based on an analytical model that can be inverted



→ **Analytical Model** → **Formalism Inversion** → **Complete error budget**

NEED FOR AN ANALYTICAL MODEL

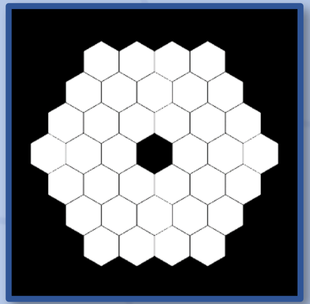
Method based on an analytical model that can be inverted



Analytical Model → Formalism Inversion → Complete error budget

- Faster than the traditional method
- Accurate enough

CONTENTS



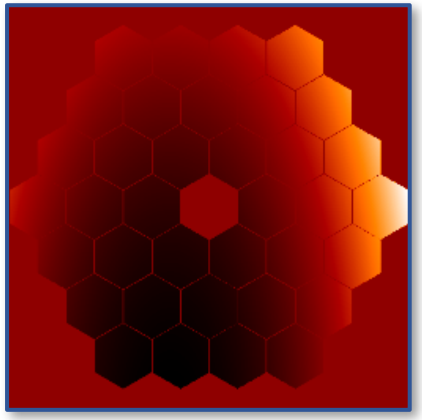
Analytical Model → Formalism Inversion → Complete error budget

- PASTIS for image generation
- PASTIS for contrast computation

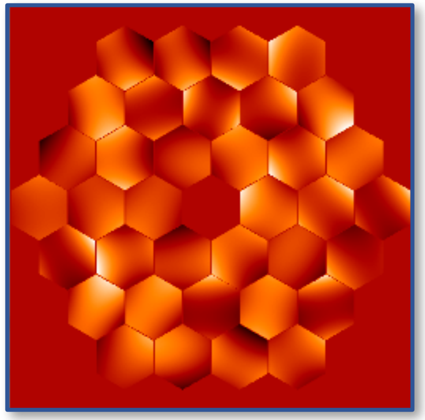
PHASE ABERRATION MODEL

Phase aberration as a sum of global and local Zernike polynomials

$$\Phi\ (\vec{r}) = $$



$$+$$



Global Zernikes

Local Zernikes

PHASE ABERRATION MODEL

Phase aberration as a sum of global and local Zernike polynomials

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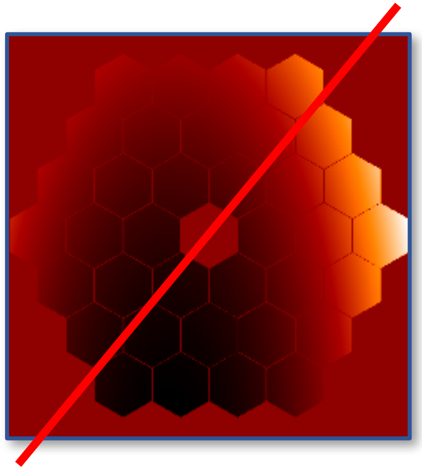
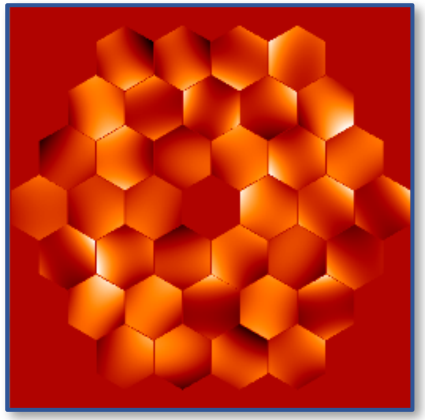


$$+$$



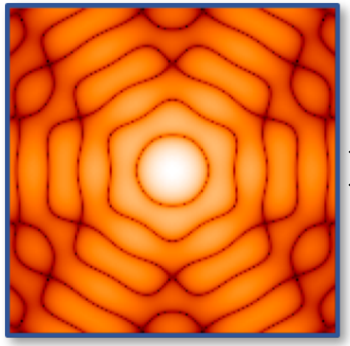
Global Zernikes

Local Zernikes

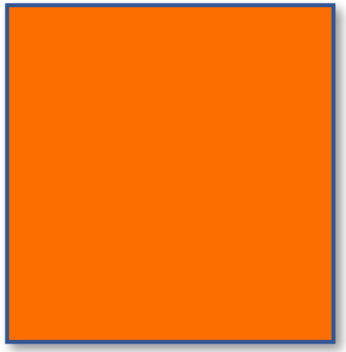
ONE SINGLE LOCAL ZERNIKE POLYNOMIAL

Intensity in focal plane in the dark region

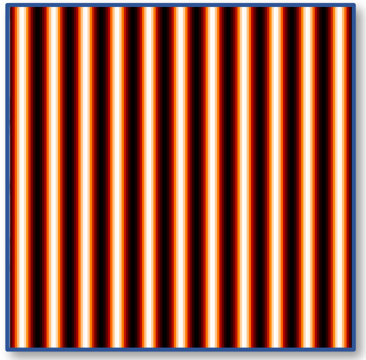
$$I(\vec{u}) =$$



$$\times \Bigg[\sum_{k=1}^{n_{seg}}$$



$$+ \sum_{k_1=1}^{n_{seg}} \sum_{k_2=1, k_2 \neq k_1}^{n_{seg}}$$



$$\Bigg]$$

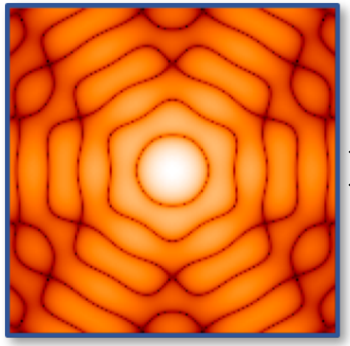
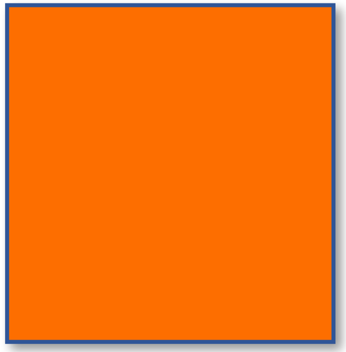
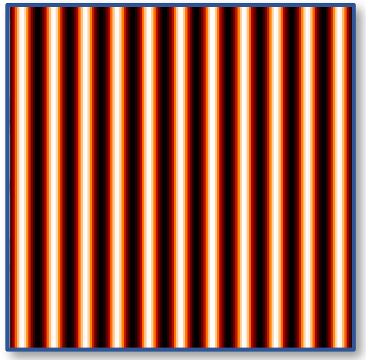
Envelope

Constant coefficient

Interference fringes between all the pairs of segments

ONE SINGLE LOCAL ZERNIKE POLYNOMIAL

Intensity in focal plane in the dark region

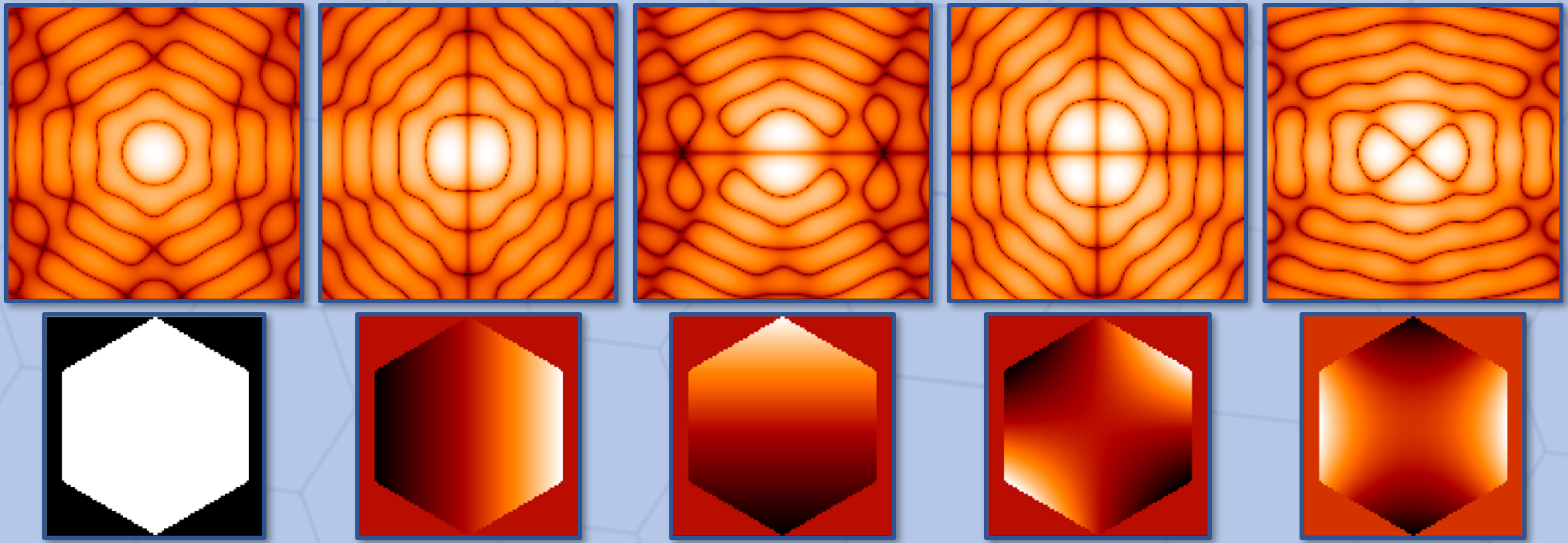
$I(\vec{u}) =$  $\times \Bigg[\sum_{k=1}^{n_{seg}}$  $+ \sum_{k_1=1}^{n_{seg}} \sum_{k_2=1, k_2 \neq k_1}^{n_{seg}}$  $\Bigg]$

Analogy to the Young experiment

$$I(\vec{u}) = \left\|\hat{P}(\vec{u})\right\|^2 \times \left[I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\vec{a}.\vec{u})\right]$$

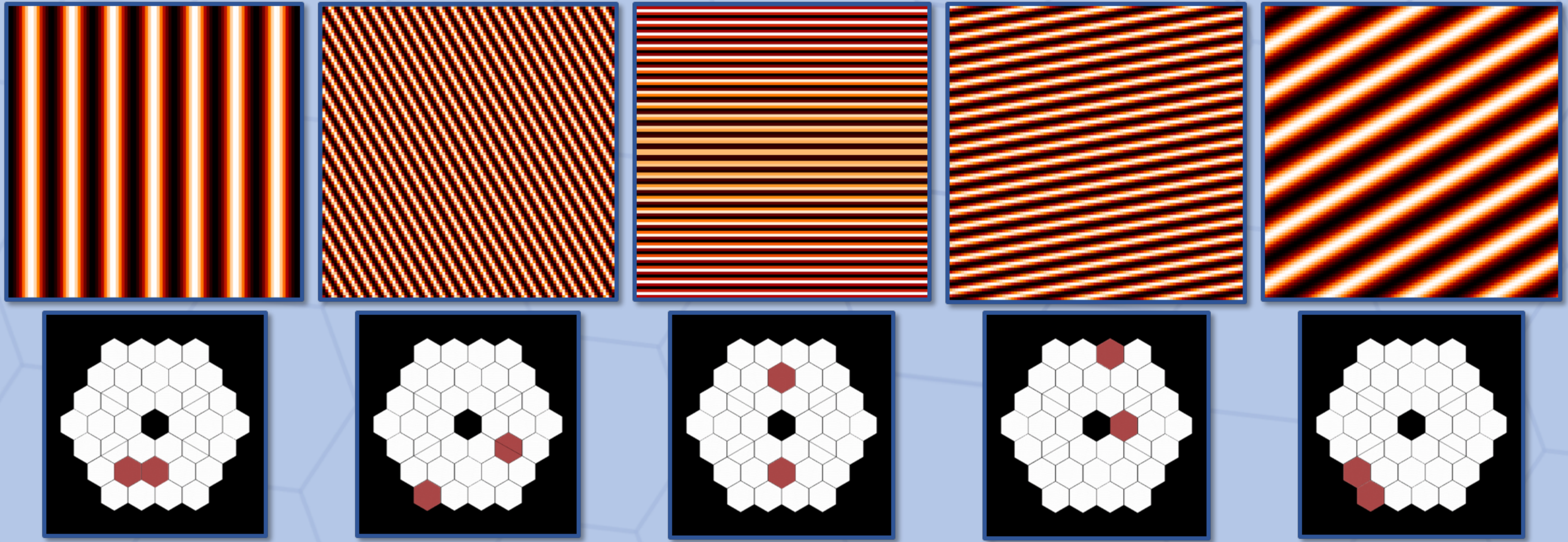
EXAMPLES OF ENVELOPES

Intensity in focal plane in log scale (top) and corresponding Zernike on one segment (bottom)



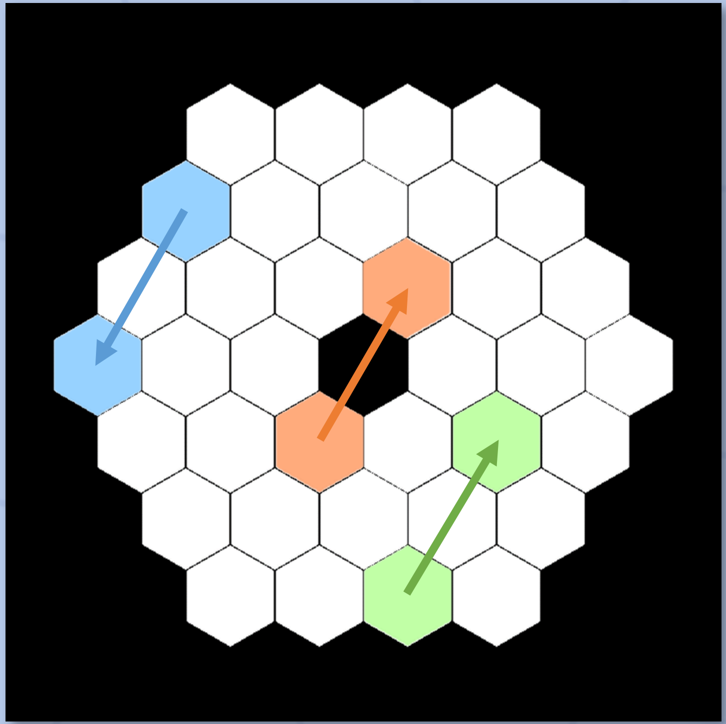
EXAMPLES OF INTERFERENCE FRINGES

Intensity in focal plane in linear scale (top) and corresponding segment pair on the pupil (bottom)



ONE SINGLE LOCAL ZERNIKE POLYNOM

« Redundant » pairs of segments generate the same interference fringes



$\overrightarrow{r_{16}} - \overrightarrow{r_{28}}$

$\overrightarrow{r_{25}} - \overrightarrow{r_{12}}$

$\overrightarrow{r_{14}} - \overrightarrow{r_{3}}$

42 oriented pairs have the same interference fringes as $\overrightarrow{r_{16}} - \overrightarrow{r_{28}}$.

These 42 pairs can be replaced by one single pair.

$(\overrightarrow{b_q})_{q\in[\![1,n_{NRP}]\!]}$: basis of non-redundant pairs of segments

n_{NRP}: number of non-redundant pairs of segments

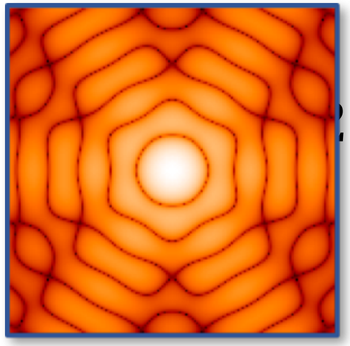
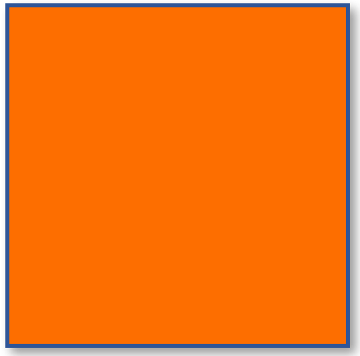
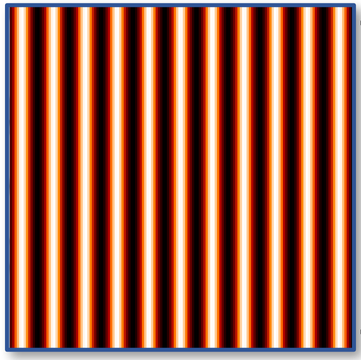
$2\times C_{36}^2 = 1260$: number of pairs generating interferences

$n_{NRP} = 63$: number of non-redundant pairs

NEW FORMALISM

Intensity in focal plane in the dark region

1260 interference fringes to sum

$I(\vec{u}) =$  $\times \left[\sum_{k=1}^{n_{seg}} \right.$  $+ \sum_{k_1=1}^{n_{seg}} \sum_{k_2=1, k_2 \neq k_1}^{n_{seg}}$  $\left. \right]$

Envelope

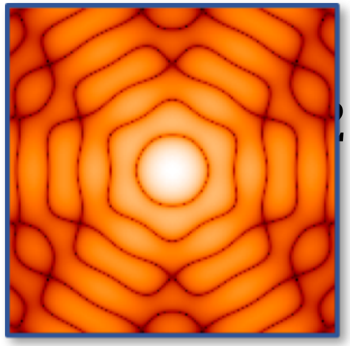
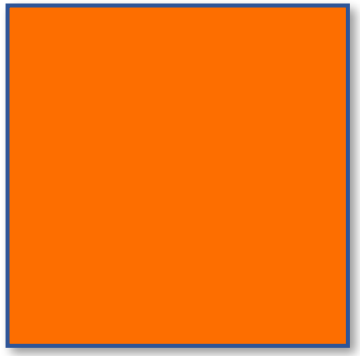
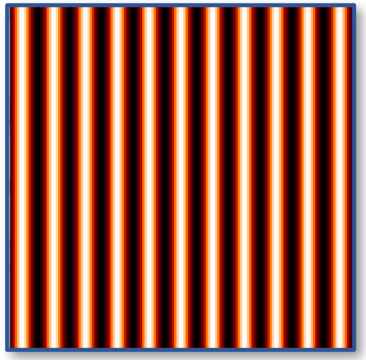
Constant coefficient

Interference fringes between all the pairs of segments

NEW FORMALISM

Intensity in focal plane in the dark region

$n_{NRP} = 63$
instead of 1260

$I(\vec{u}) =$  $\times \Bigg[\sum_{k=1}^{n_{seg}}$  $+ \sum_{q=1}^{n_{NRP}}$  $\Bigg]$

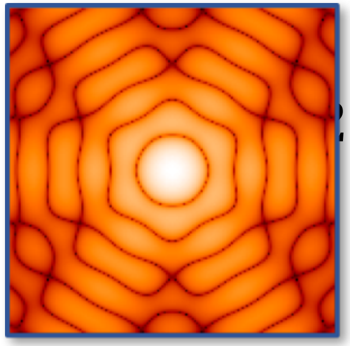
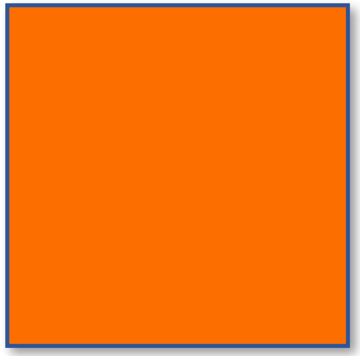
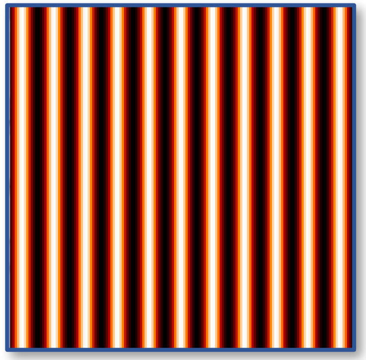
Envelope

Constant coefficient

Interference fringes between all the <u>NON-REDUNDANT</u> pairs of segments

NEW FORMALISM

Intensity in focal plane in the dark region

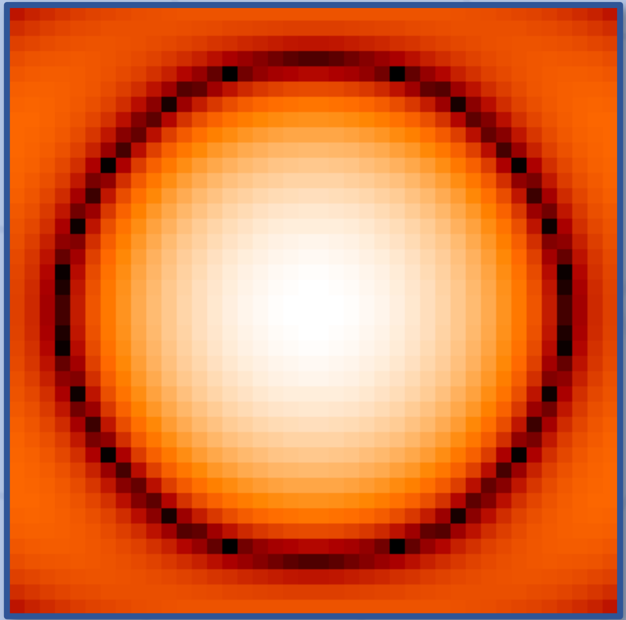
$I(\vec{u}) =$  $\times \left[\sum_{k=1}^{n_{seg}}\right.$  $+ \sum_{q=1}^{n_{NRP}}$  $\left.\right]$

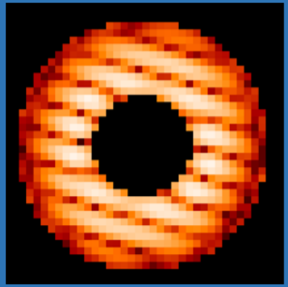
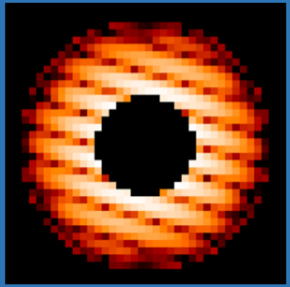
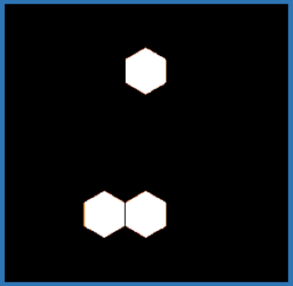
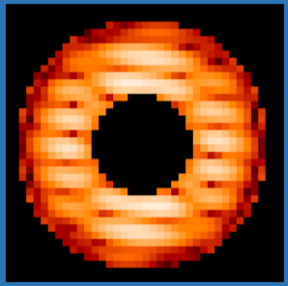
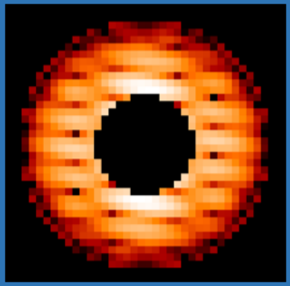
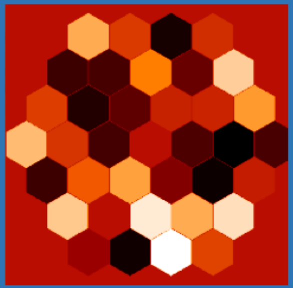
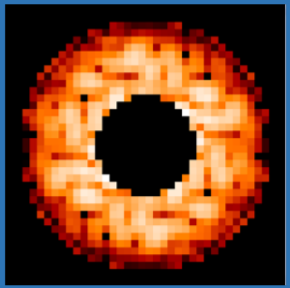
▶ **Pair-based Analytical model for Segmented Telescopes Imaging from Space (PASTIS) for image generation**

END-TO-END SIMULATION VS MODEL

Piston case

Piston envelope

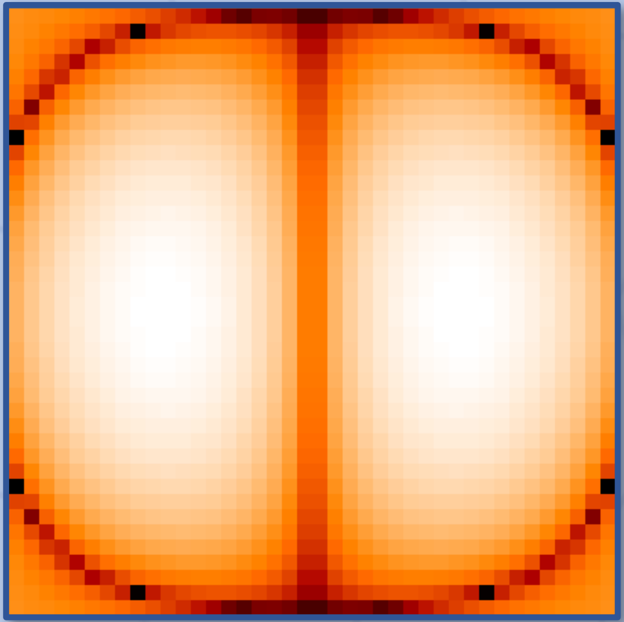


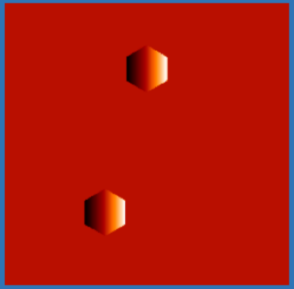
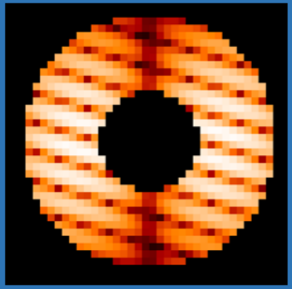
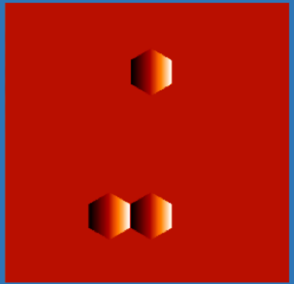
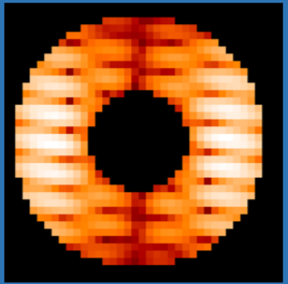
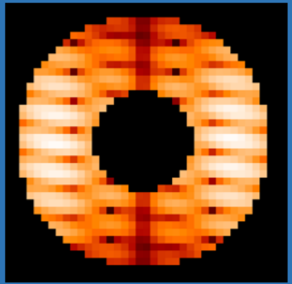
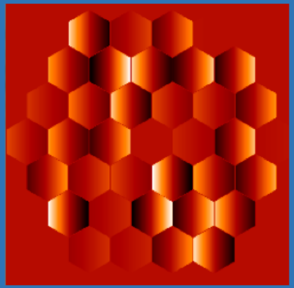
Phase ϕ	End-to-end simulation	Analytical model	Correlation
			0.71
			0.76
			0.64

END-TO-END SIMULATION VS MODEL

Tip case

Tip envelope

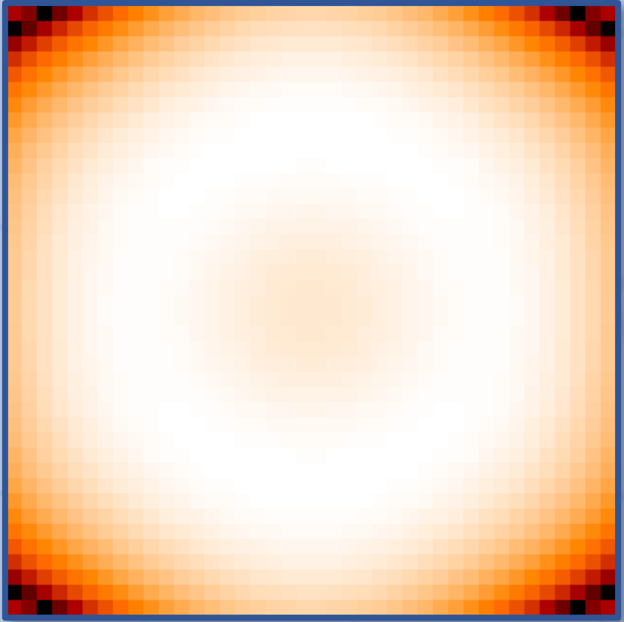


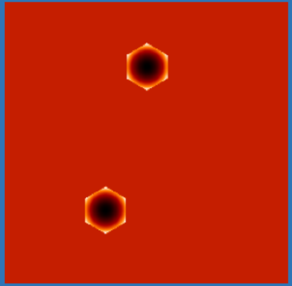
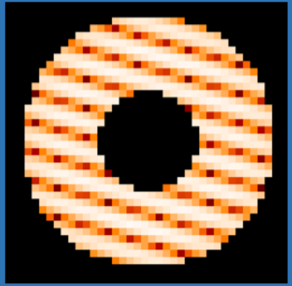
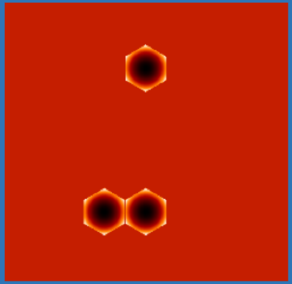
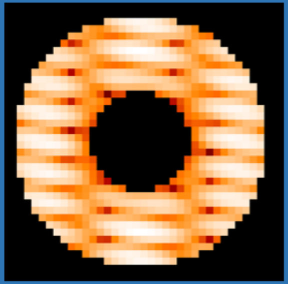
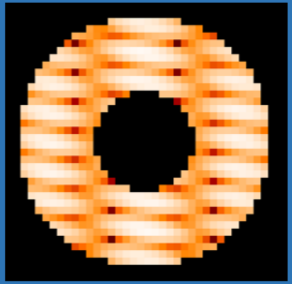
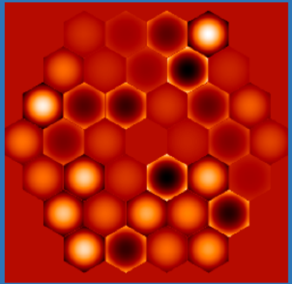
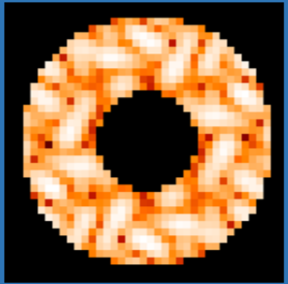
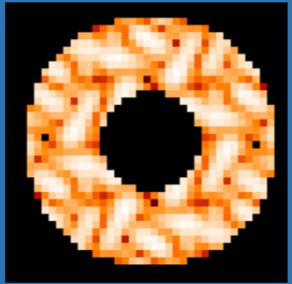
Phase ϕ	End-to-end simulation	Analytical model	Correlation
			0.93
			0.99
			0.91

END-TO-END SIMULATION VS MODEL

Focus case

Focus envelope

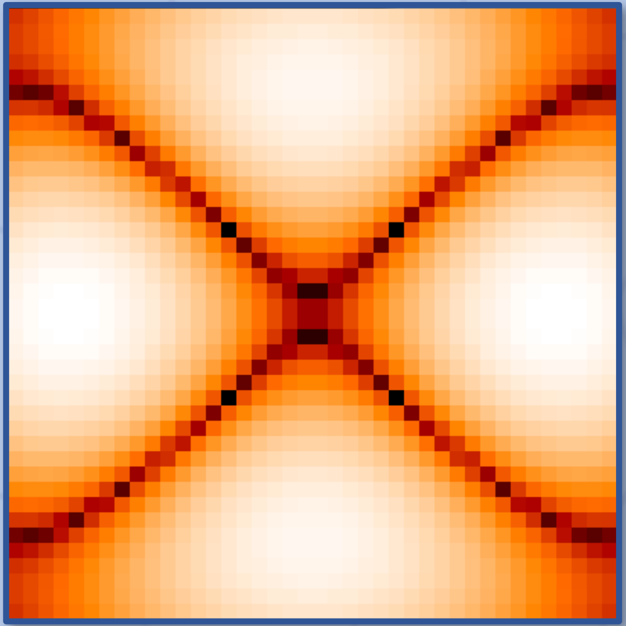


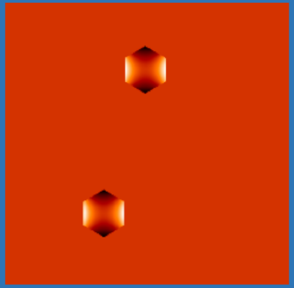
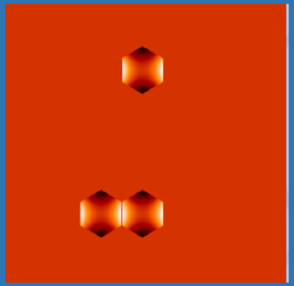
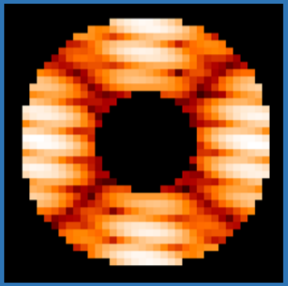
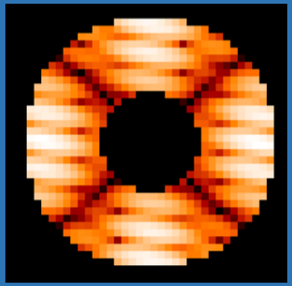
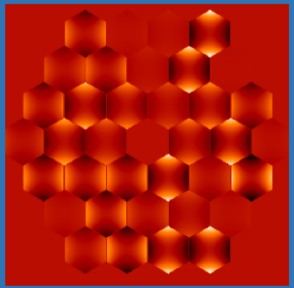
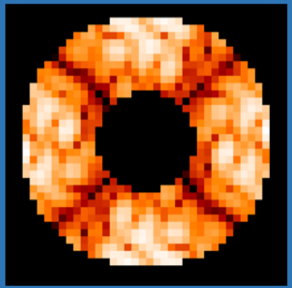
Phase ϕ	End-to-end simulation	Analytical model	Correlation
			0.89
			0.93
			0.92

END-TO-END SIMULATION VS MODEL

0°-astigmatism case

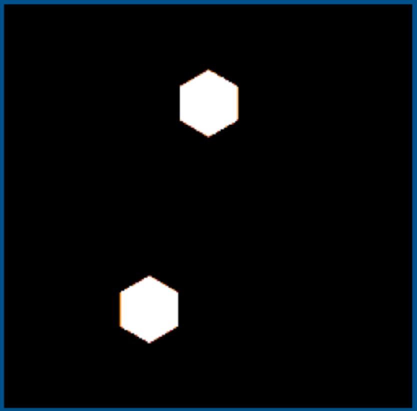
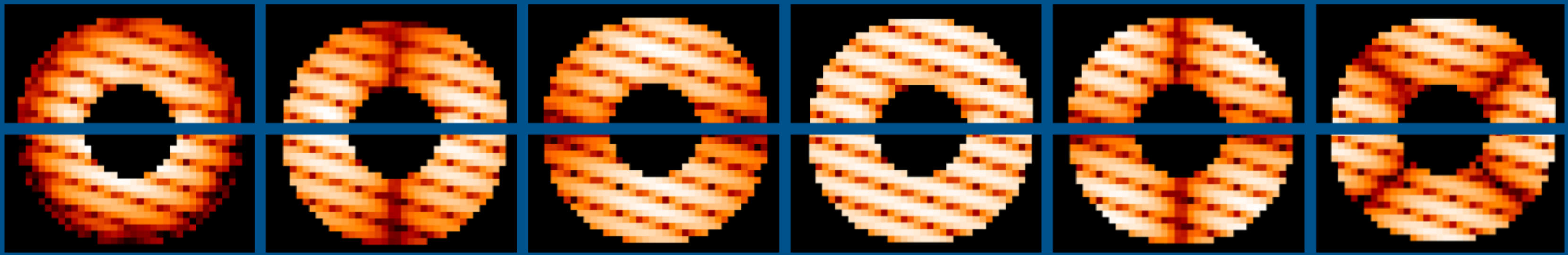
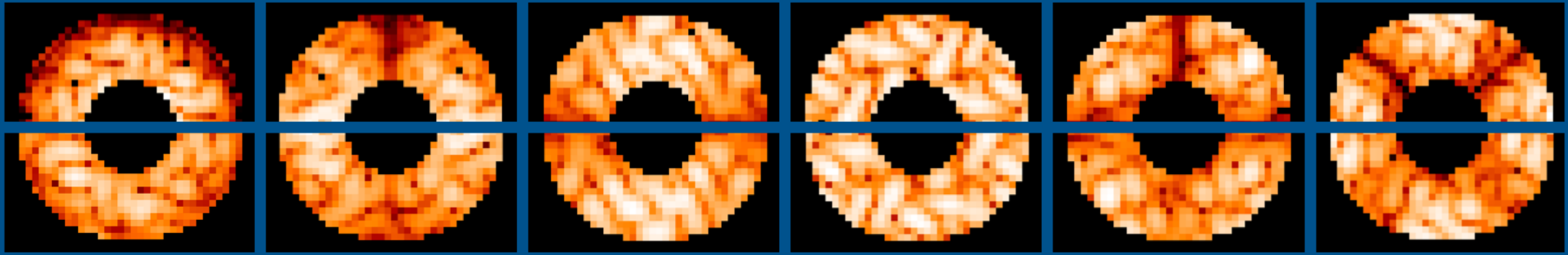
0°-Astigmatism envelope



Phase ϕ	End-to-end simulation	Analytical model	Correlation
			0.99
			1.00
			0.96

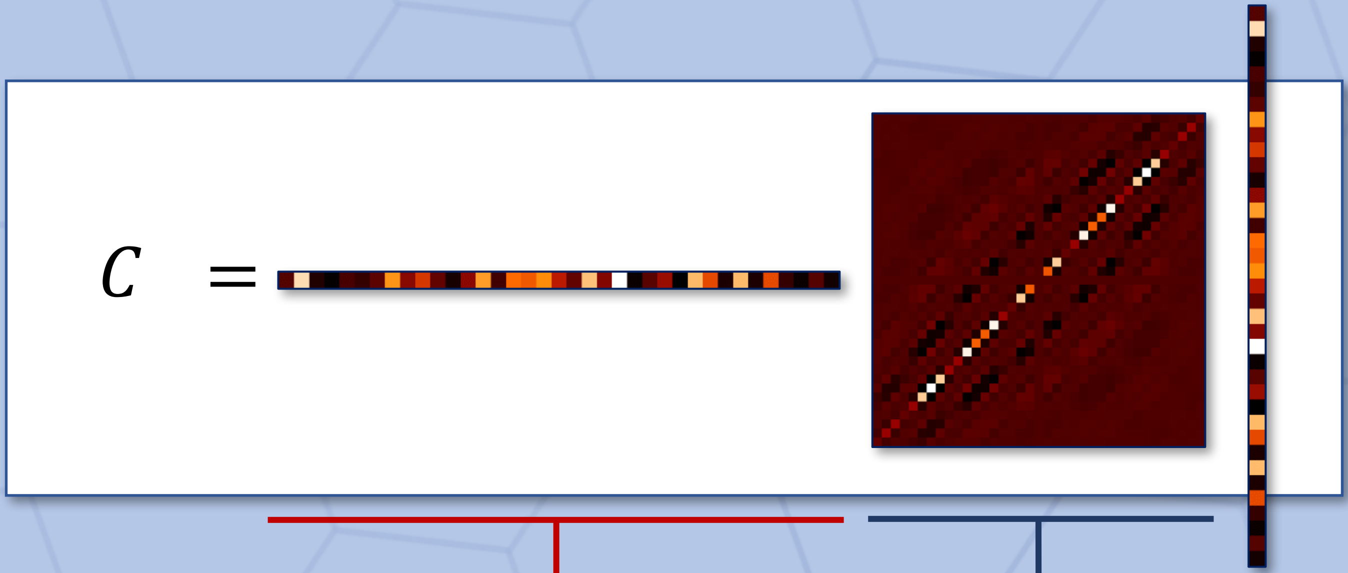
END-TO-END SIMULATION VS MODEL

Other cases

Aberration → ↓Affected segments							
							E2E
							PASTIS
	0.71	0.93	0.90	0.89	0.99	0.99	COR
							E2E
							PASTIS
	0.64	0.91	0.92	0.92	0.97	0.96	COR

NEW FORMALISM

Contrast in focal plane in the dark region

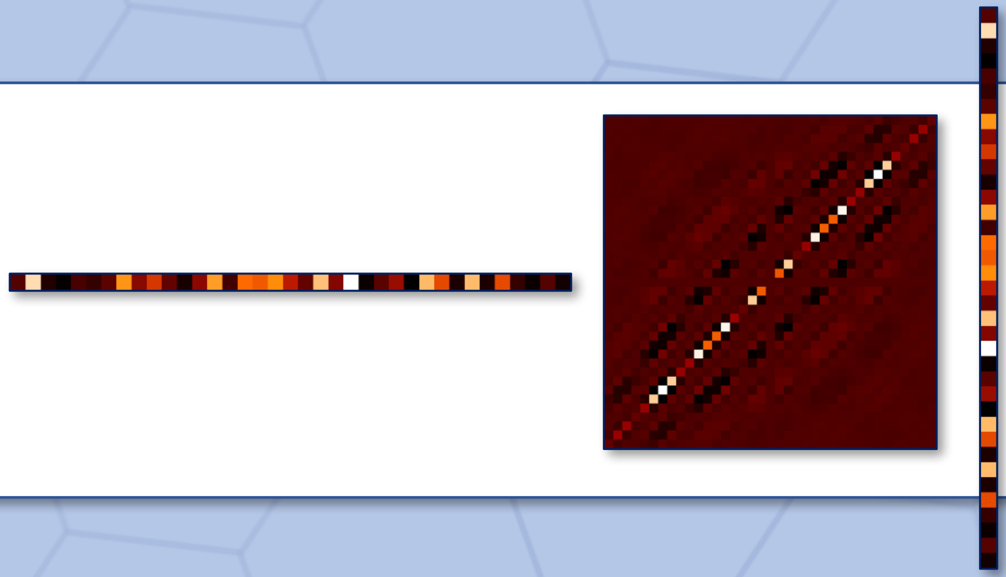


n_{seg}-long vector with the Zernike coefficients
→ Depends ONLY on the coefficients

Matrix M
→ Depends ONLY on the system and type of aberration:
- Zernike polynomial
- Segment structure
- Coronagraph

Contrast in focal plane in the dark region

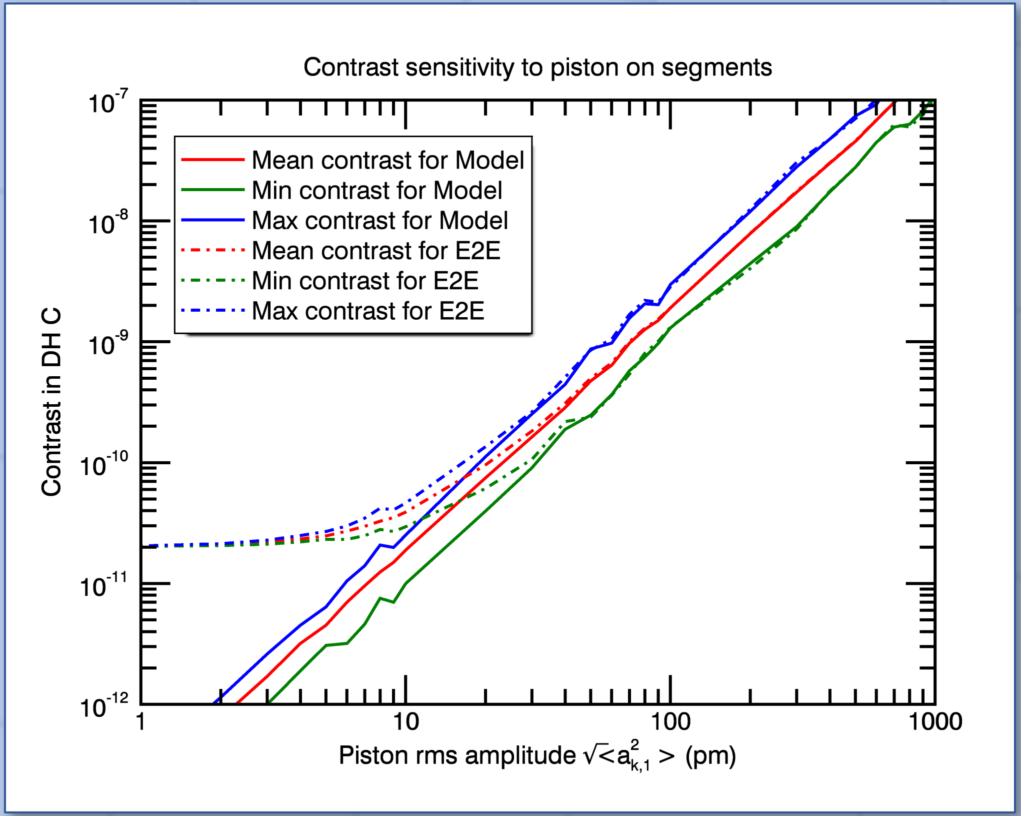
$$C =$$



▶ **Pair-based Analytical model for Segmented Telescopes Imaging from Space (PASTIS) for contrast computation**

END-TO-END SIMULATION VS MODEL

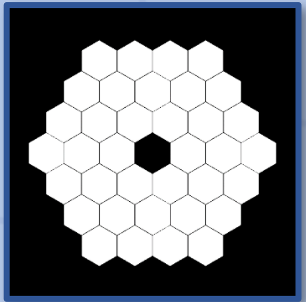
Results with PASTIS for contrast computation



- In the second regime, error between PASTIS and E2E simulation is ~3%
- Ratio of computation times between PASTIS and E2E simulation is ~10^7

NEED FOR AN ANALYTICAL MODEL

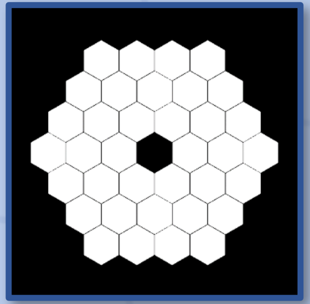
Method based on an analytical model that can be inverted



Analytical Model → Formalism Inversion → Complete error budget

- Faster than the traditional method ☺
- Accurate enough ☺

CONTENTS



Analytical Model → Formalism Inversion → Complete error budget

NUMERICAL APPLICATION STRATEGY

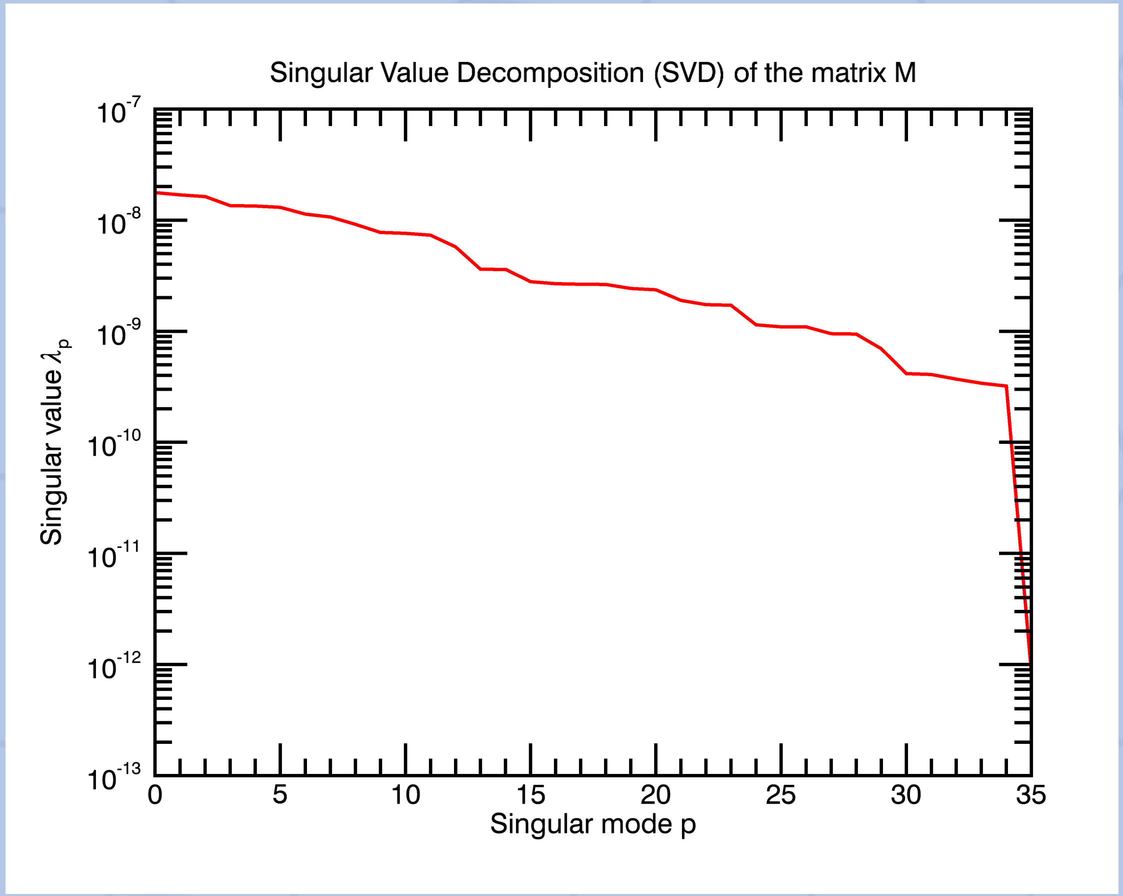
CORONAGRAPH+ OPTICAL SYSTEM
$C \leq 10^{-6}$

↓

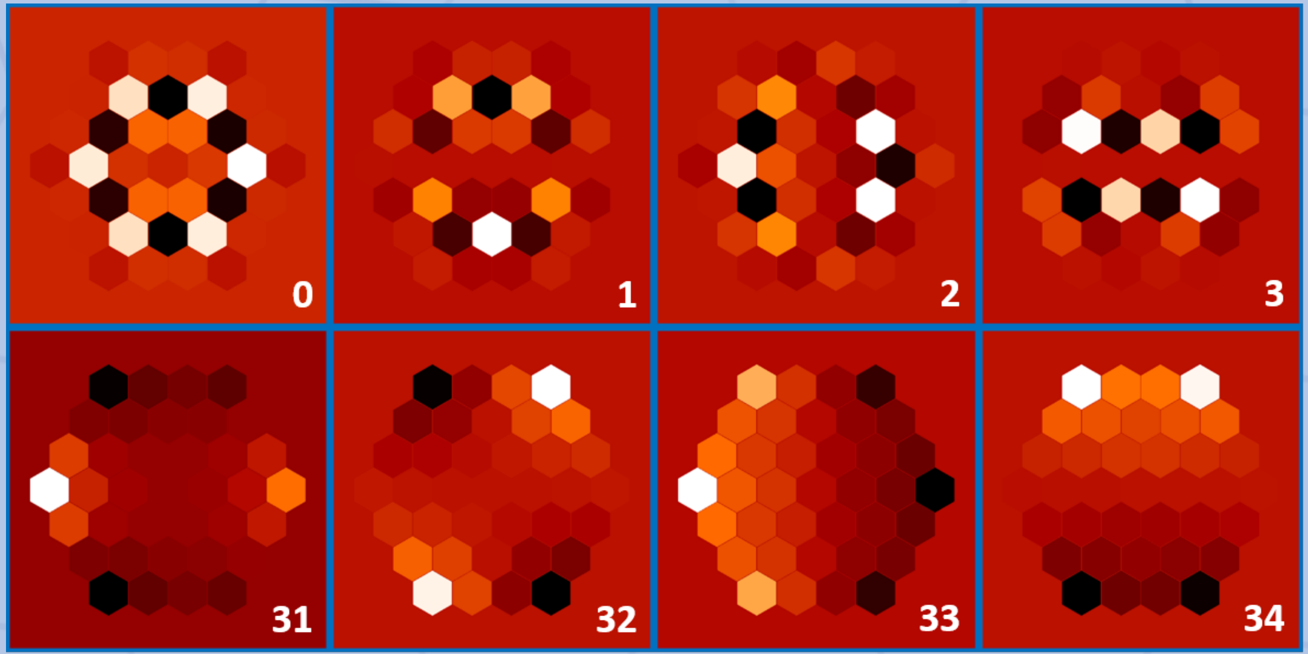
WAVEFRONT CONTROL
$C \leq 10^{-10}$

EIGEN MODES ON THE SEGMENTED MIRROR

Singular Value Decomposition of the matrix M in the piston case



Eigen values



A few eigen modes

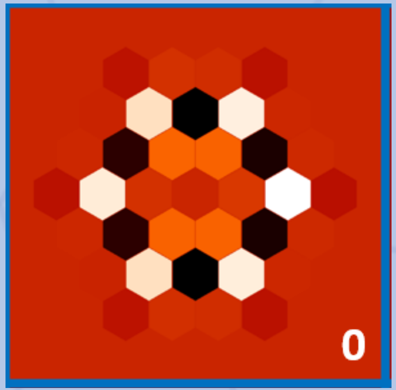
PASTIS INVERSION

Hypotheses and conclusion for PASTIS inversion

"Worst" phase

$C = 10^{-6}$

$= \sigma_0$



$+ \cdots + \sigma_{34}$

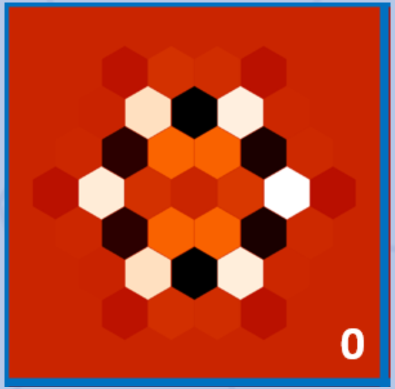
Eigen modes

Maximum contributions to each eigen mode
= Constraints

PASTIS INVERSION

Hypotheses and conclusion for PASTIS inversion

$=\sigma_0$



$+ \cdots + \sigma_{34}$

"Worst" phase
$C = 10^{-6}$

$$C_0 = \frac{10^{-6}}{35}$$

$$C_{34} = \frac{10^{-6}}{35}$$

PASTIS INVERSION

Hypotheses and conclusion for PASTIS inversion

Known

- The phase can be projected on the modes' basis — $\Phi = \sum \sigma_i \Phi_i$
- The contrast generated by the phase is the sum of the contrasts generated by its projection on each mode — $C = \sum C_i$

Conclu-sion

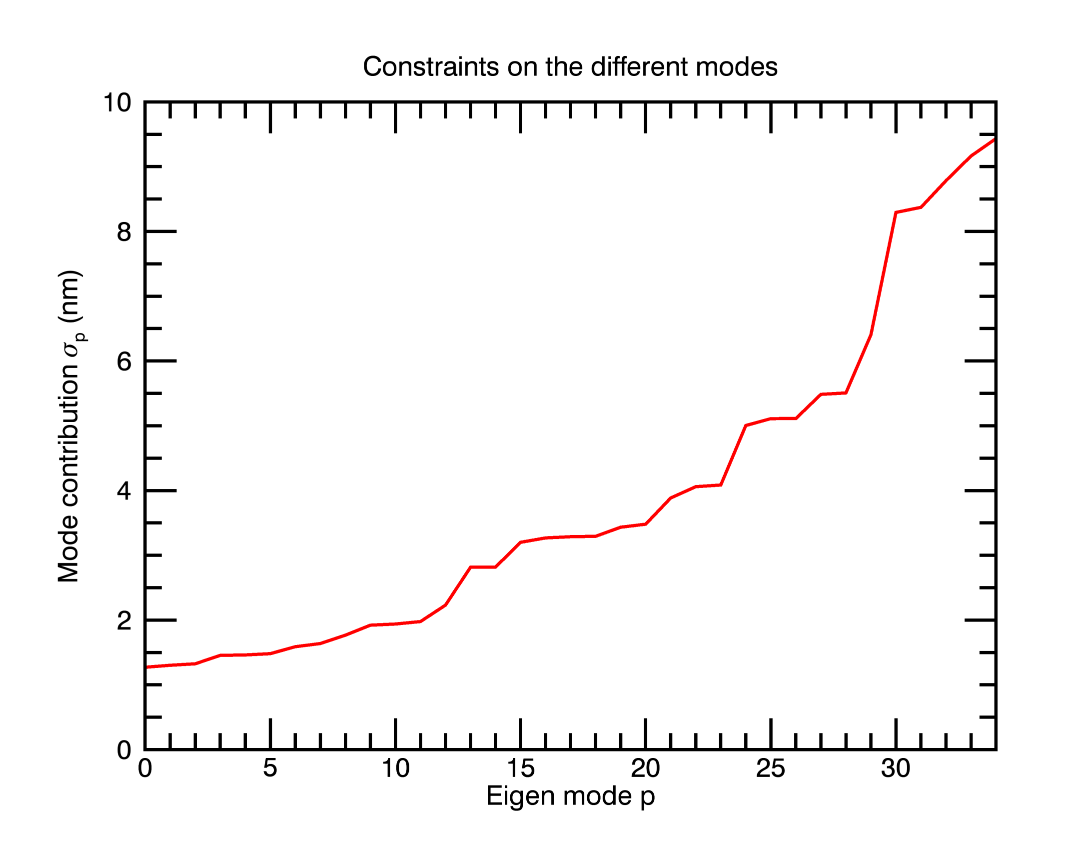
- The contribution to each mode is — $\sigma_i = \sqrt{\frac{C_i}{\lambda_i}}$

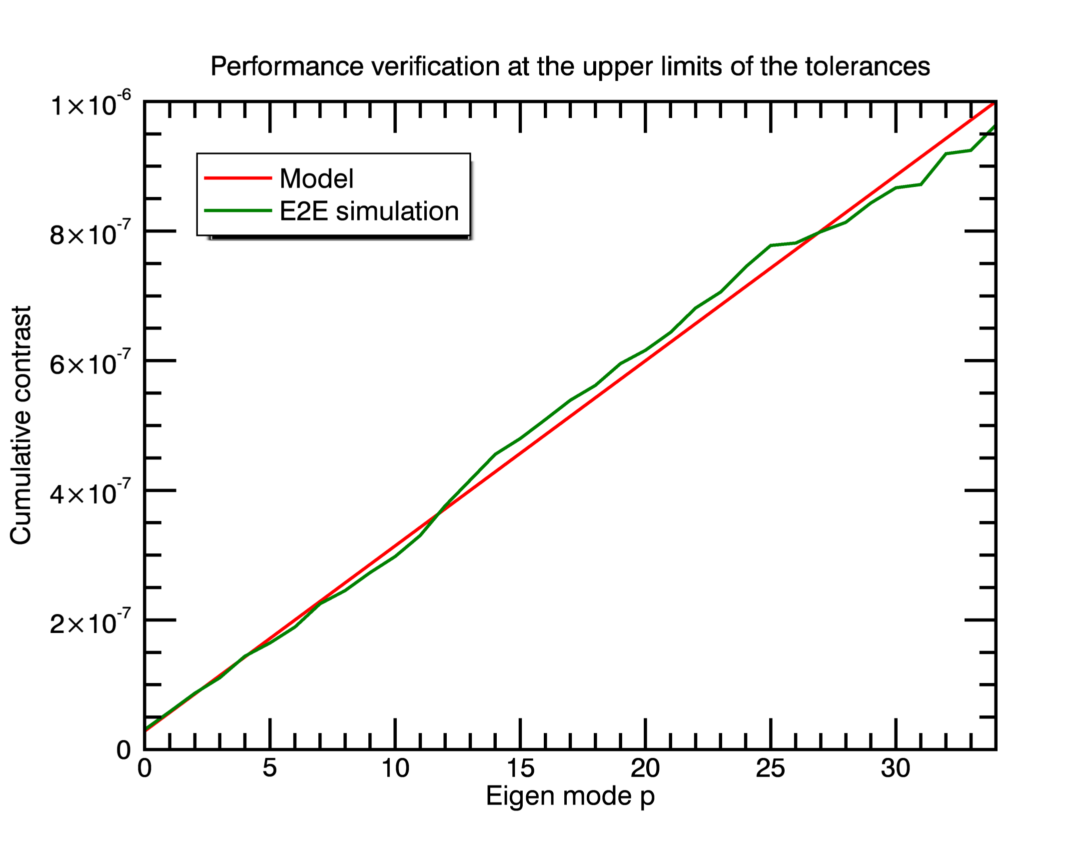
Free

- The projection on each mode contributes equally to the contrast — $C_i = C_0$
- Numerical application — $C = 10^{-6}$

APPLICATION TO ERROR BUDGETING

Constraints on the different modes for a target contrast of 10^{-6}





PASTIS INVERSION ON STABILITY

Hypotheses and conclusion for PASTIS inversion

Known

- The uncertainty on the contrast(s) — $C^r = C \pm \Delta C$

  $C_i^r = C_i \pm \Delta C_i$
- The contrast generated by the phase is a function of the contrasts generated by its projection on each mode — $\Delta C = \sqrt{\sum \Delta C_i^2}$

Conclu-sion

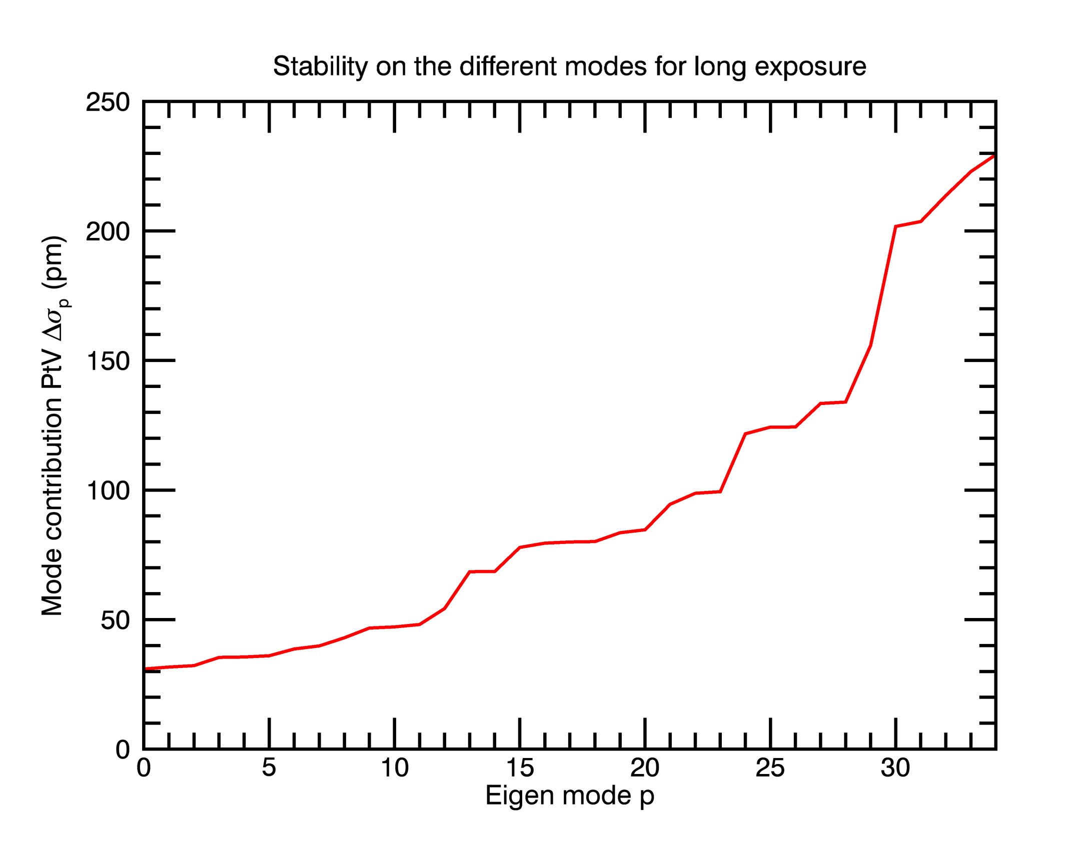
- The contribution to each mode is — $\Delta \sigma_i = \sqrt{\frac{\Delta C_i}{\lambda_i}}$

Free

- The projection on each mode contributes equally to the contrast — $\Delta C_i = \Delta C_0$
- Numerical application — $\Delta C = 10^{-10}$

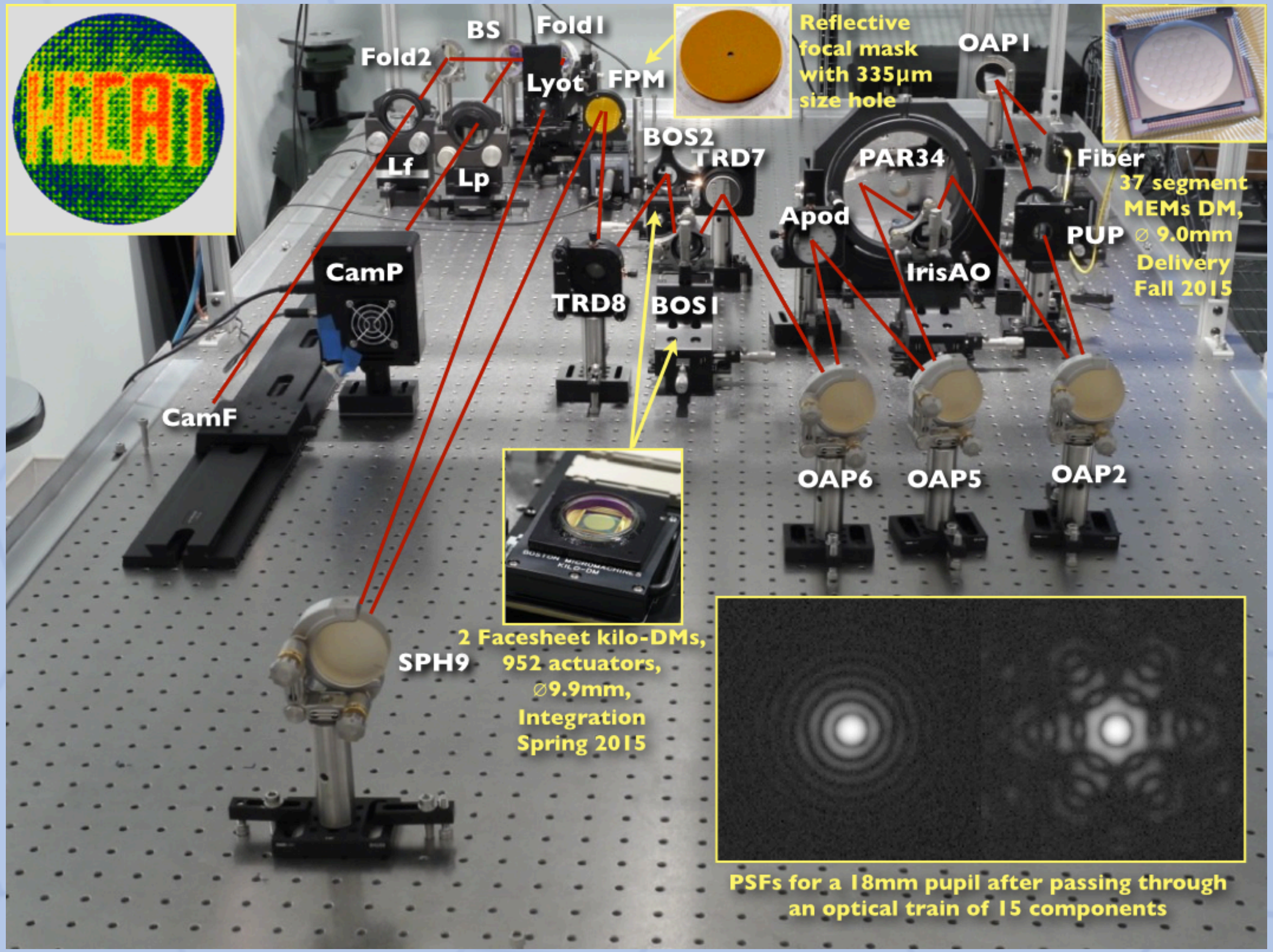
APPLICATION ON STABILITY CONSTRAINTS

Constraints on the different modes for a stability on the contrast of 10^{-10}



HiCAT

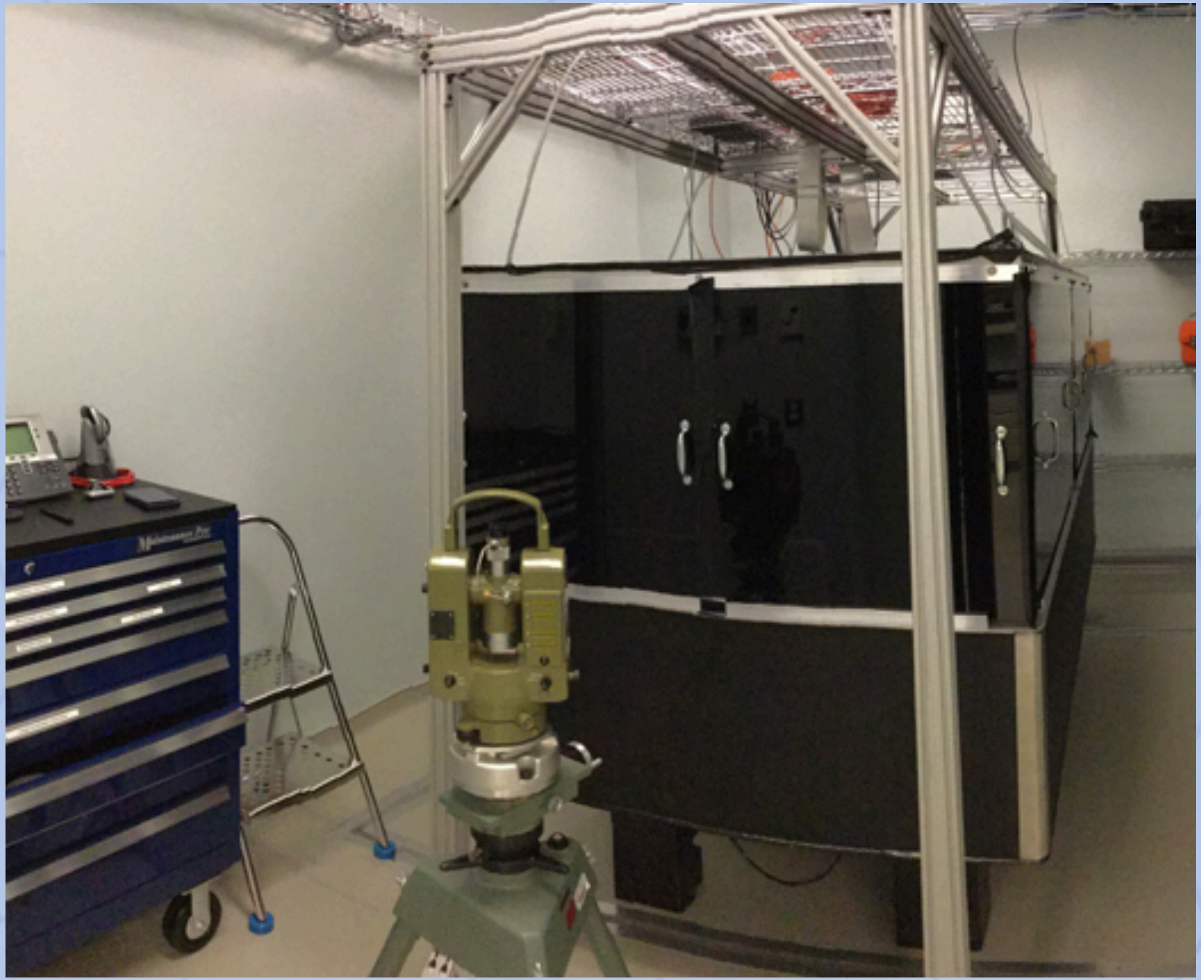
High-contrast imager for Complex Aperture Telescopes



- **Simulation of a segmented telescope:** segmented mirror + mask for central obstruction and spiders
- **Apodized Lyot Coronagraph** Apodizer + Focal Plane Mask + Lyot Stop
- **Wavefront Sensing:** phase retrieval/deformable mirror
- **Wavefront Control:** two deformable mirrors

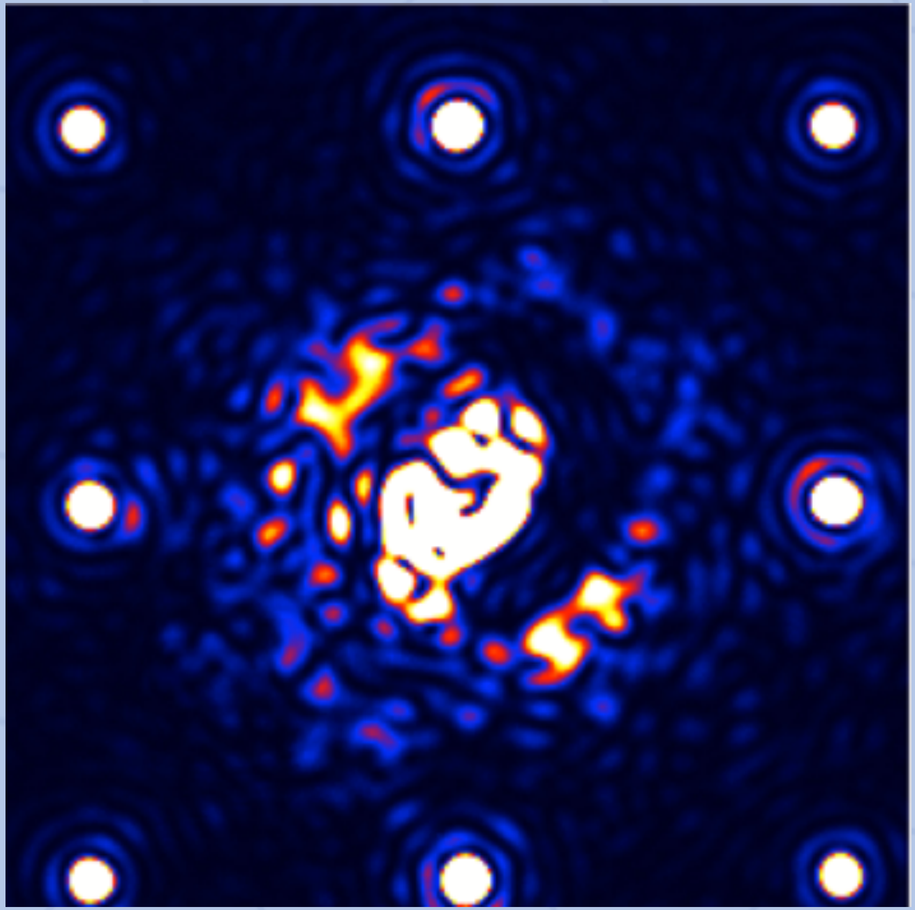
HiCAT

High-contrast imager for Complex Aperture Telescopes



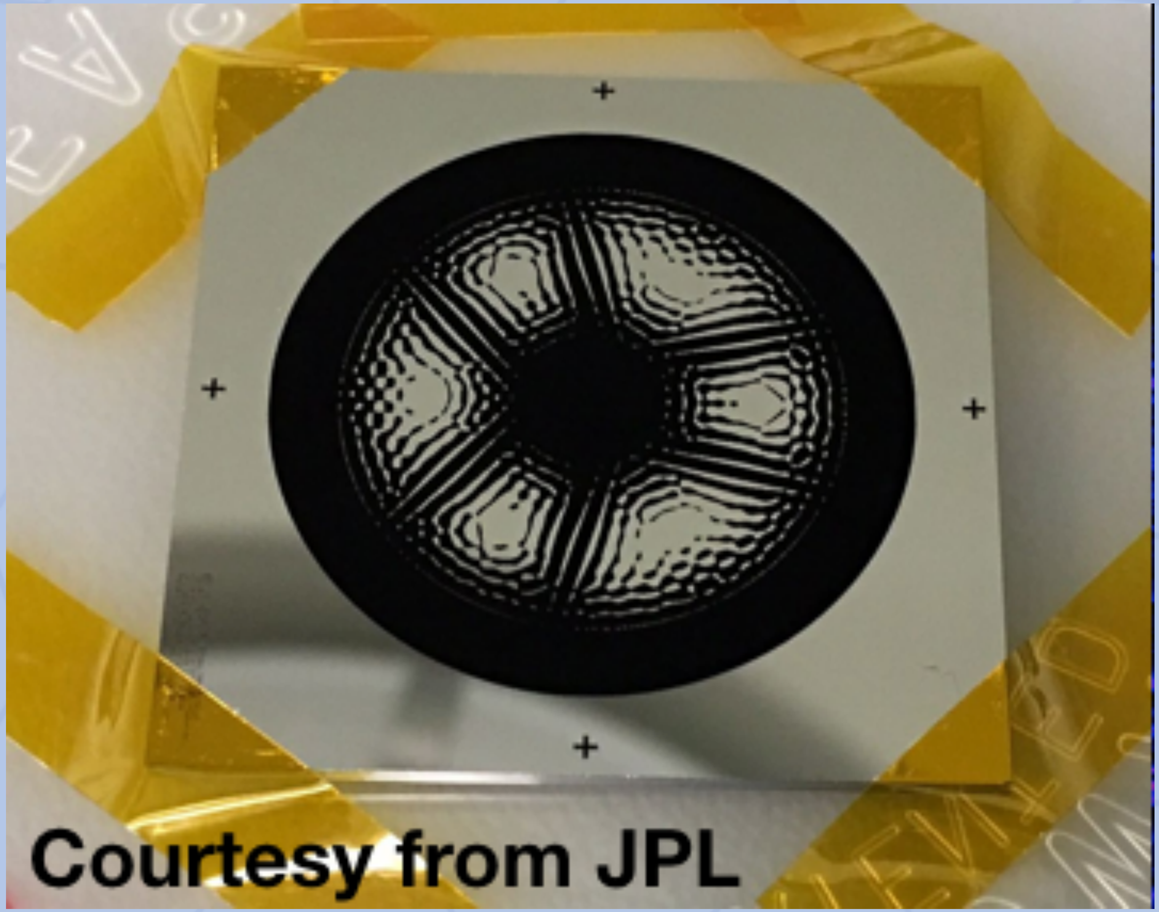
- Baffling
- Stabilized mounting
- Room and box controlled in temperature, pressure, humidity
- Remote control

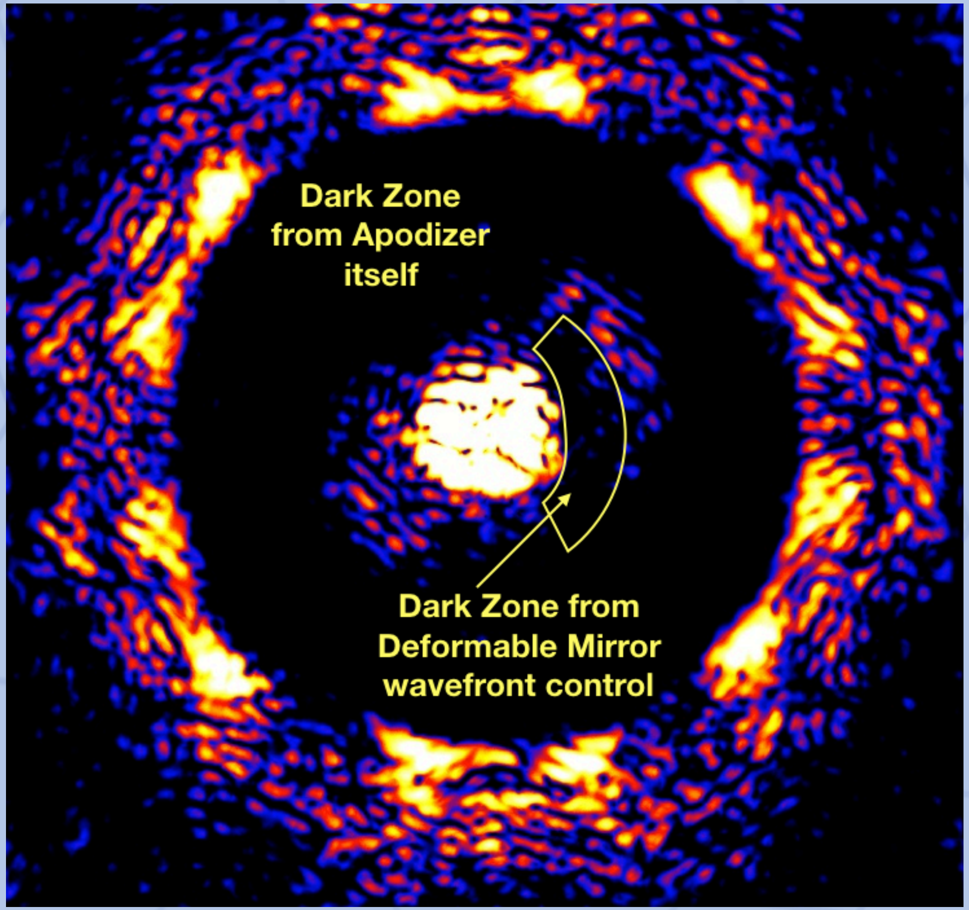
Monolithic pupil + no apodizer + Speckle Nulling



HiCAT

Monolithic pupil + WFIRST apodizer + Speckle Nulling



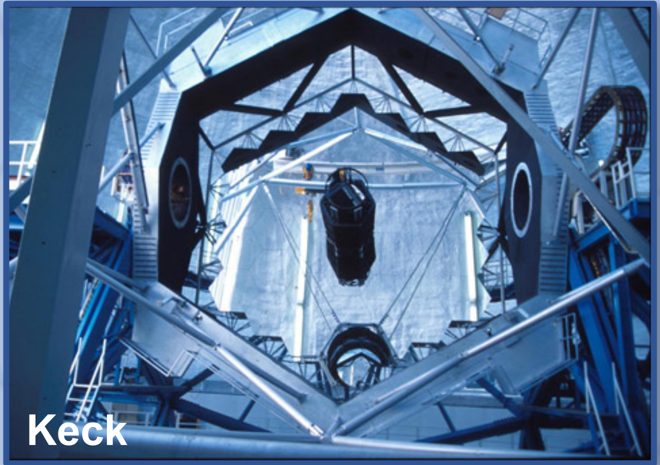


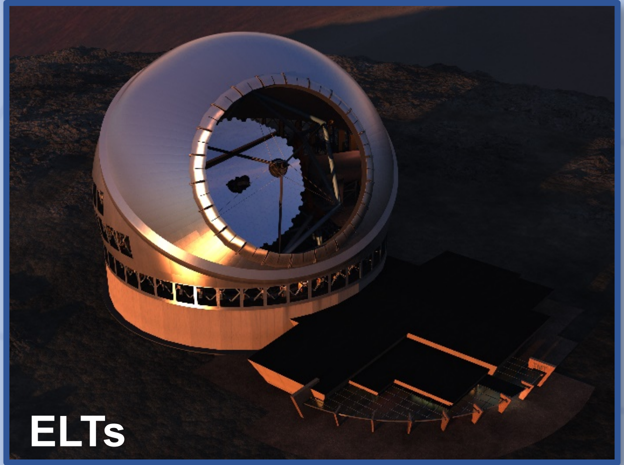
LUVOIR-like apodizer (coming soon)

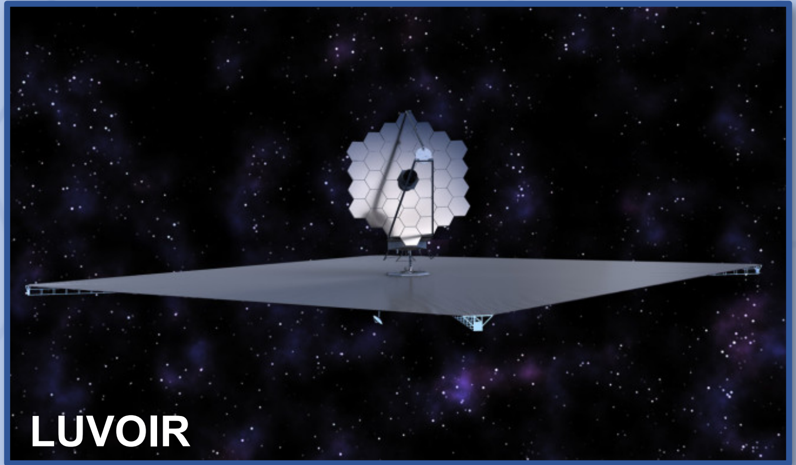
CONCLUSIONS AND PERSPECTIVES

Validation and interest of the analytical model

- Very close correlation between end-to-end simulation and analytical model: error lower than 3% in contrast computation
- Inversion of the formalism to provide the upper-limit Zernike coefficients that verify a defined contrast to provide a complete error budget
- Better understanding of the impact of the segments on the contrast: optimal backplane structure…
- Easily adaptable to all segmented pupils, even with non-hexagonal segments:


Keck


ELTs

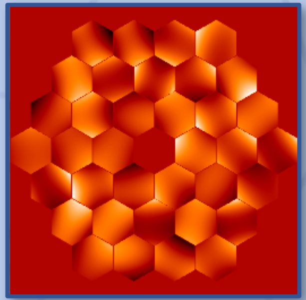

LUVOIR

CONCLUSIONS AND PERSPECTIVES

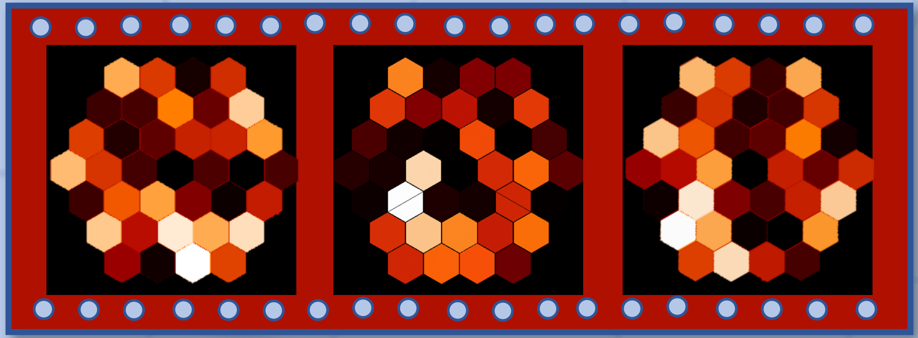
To go further…

Three main developments of PASTIS:

- **Generalization to combine multiple local Zernike polynomials, that has a similar form than the current case, with a simple multiplication between:**
  - **an envelope (segment-shape- and Zernike polynomial-dependant)**
  - **a finite sum of interference fringes between each pair of pupil segments**



- **Application to vibrations and resonant modes of the segments**



- **Application to broadband light**

THANK YOU
FOR YOUR ATTENTION!

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