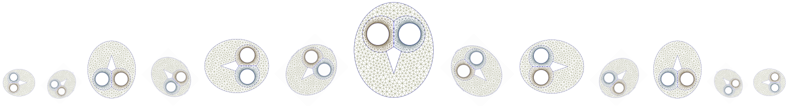


May 24th, 2018

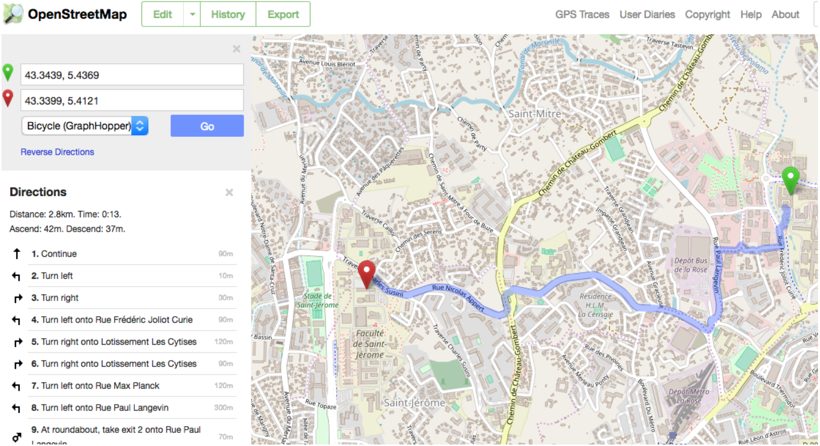
Finite element modeling of nanophotonic structures - Applications
LAM Seminar

Guillaume Demésy[1] (+ many people from Institut Fresnel)

guillaume.demesy@fresnel.fr

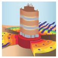
[1] Aix-Marseille Université, CNRS, Centrale Marseille, **Institut Fresnel** UMR 7249, 13013 Marseille, France

Foreword: Fresnel on the map



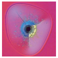
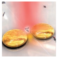
Foreword: Fresnel by themes

ELECTROMAGNÉTISME
MÉTAMATÉRIAUX



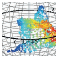

• Modèles et fondamentaux en électromagnétisme (Analogie micro-onde, Nouvelles approches d'homogénéisation, Etude des effets de la dispersion, Non-linéarités spatiales)
• Méthodes numériques (Méthode intégrale de volume et forces optiques, Méthode des éléments finis, Méthode intégrale de surface, Méthode Monte Carlo et milieux diffusants)
• Réseaux de diffraction et fibres micro-structurées (Analyse d'effets physiques, Fltres à résonance de mode guide, Fibres optiques micro-structurées, Conception de composants optiques)
• Métamatériaux, invisibilité et protection (métamatériaux en optique et micro-ondes, métamatériaux en acoustique et mécanique, Protections hydrodynamique et sismique, Chaleur et mimétisme, Application des métamatériaux au bio-médical)

NANOPHOTONIQUE
COMPOSANTS OPTIQUES

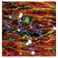
• Interactions lumière-matière aux échelles nanométriques (Aspects fondamentaux de la mécanique quantique, Emission exaltée par des nanoantennes, Contrôle nano-optique de la directivité d'émission)
• Thermoplasmonique et nano-résonateurs optiques (Absorption de lumière et thermoplasmonique, Théories multipolaires et modales, Nanophotonique sur particules diélectriques)
• Couches minces optiques (Filtres optiques interférentiels à hautes performances, Composants et concepts innovants, Métrologie extrême et diffusion lumineuse, nouveaux instruments et procédés)
• Interaction laser-matière aux forts flux (Etude des processus physiques de l'interaction laser-matière aux forts flux, Composants optiques pour lasers de puissance, Procédés laser)

TRAITEMENT DE L'INFORMATION
ONDES ALÉATOIRES

• Polarisation et cohérence optique (Milieux désordonnés et aléatoires, Optique statistique, Instrumentation…)
• Télécommunications et traitement d'antenne (Réseaux de capteurs, Systèmes de communication optique sans fil, Cryptographie quantique…)
• Traitements et modèles pour la Télédétection (Interactions onde / surface océanique, Imagerie hyper-spectrale, Imagerie SAR polarimétrique et inter-férométrique, Imagerie sous-marine…)
• Eléments méthodologiques pour l'image et le signal multi-dimensionnel (Segmentation et poursuite pour les images bruitées, Biométrie et reconnaissance de gestes, Imagerie médicale, Segmentation ultra-rapide…)

IMAGERIE AVANCÉE
VIVANT




• Instrumentation (Techniques de microscopie optique, Fibres optiques pour la spectroscopie et l'endoscopie, Instrumentation et caractérisation en hyperfréquence, Autres développements en instrumentation…)
• Reconstruction numérique (Microscopie tomographique diffractive optique, Tomographie micro-ondes, Tomographie photo-acoustique quantitative, Microscopie de fluorescence à illumination structure, Imagerie X cohérente, Caractérisation multiéchelle)
• Etude du vivant (Imagerie des tissus, Imagerie des structures biologiques à l'échelle cellulaire, Imagerie quantitative de phase et de température en milieu cellulaire, Nouvelles sondes moléculaires et inorganiques pour l'imagerie biologique)

Foreword: Picking an appropriate model

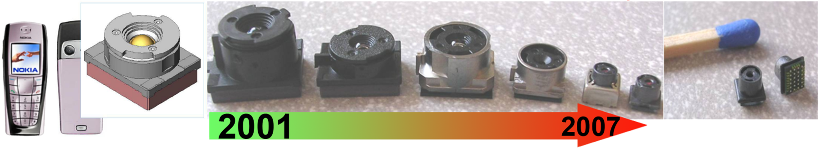
$$\ldots \xrightarrow{\text{homogeneous } \varepsilon_r, \mu_r} \text{Electromagnetism} \xrightarrow{\text{paraxial}} \text{Beam/Fourier Optics} \longrightarrow \text{Ray Optics}$$

Outline

Drastic reduction of the pixel pitch in CMOS “cheap” sensors

300 000
6 µm

3 000 000
1.75 µm



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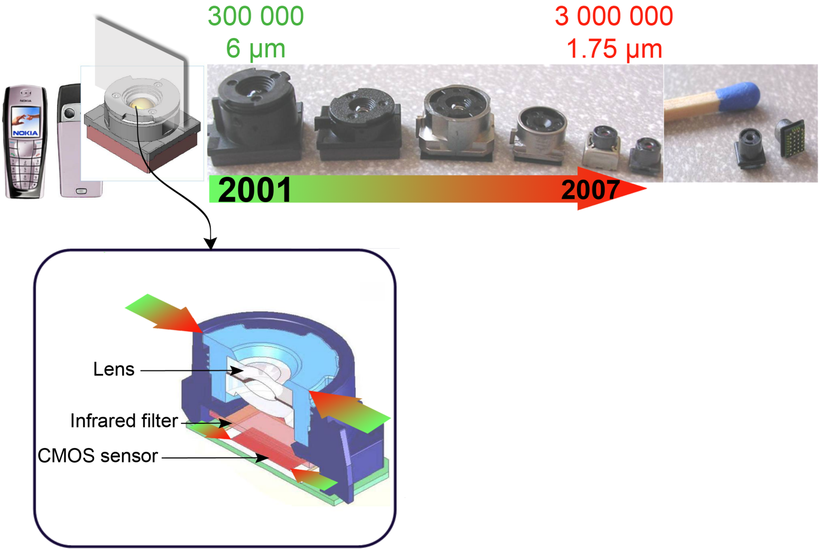

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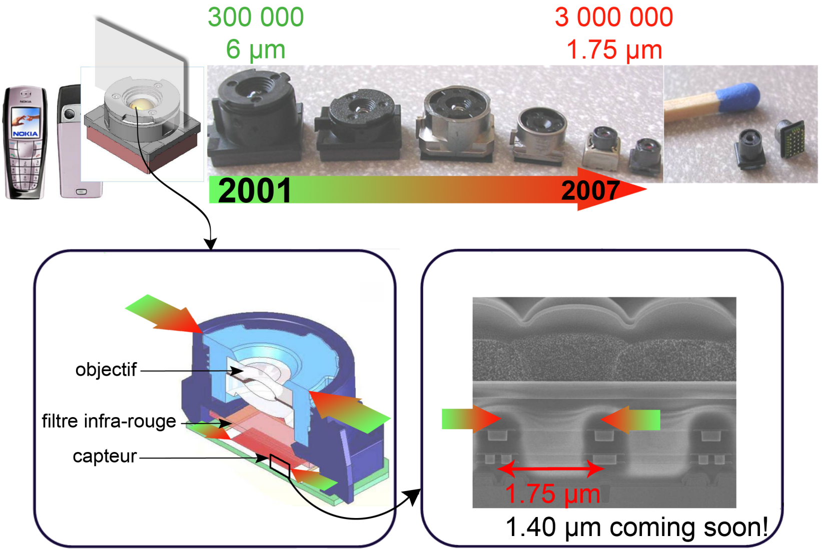
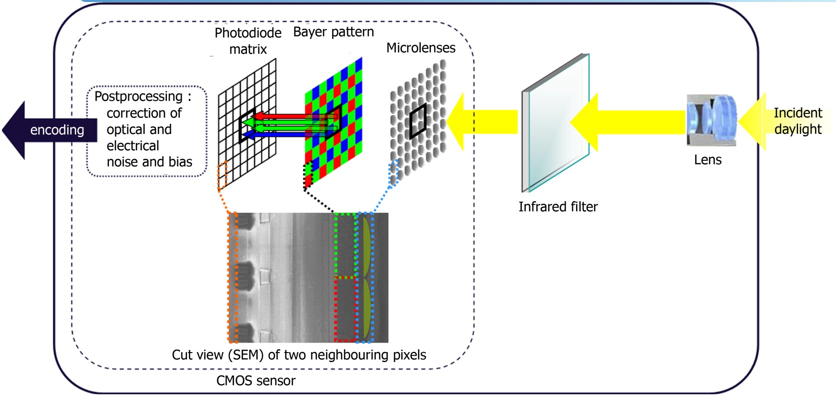


Constitutive elements of a complete camera



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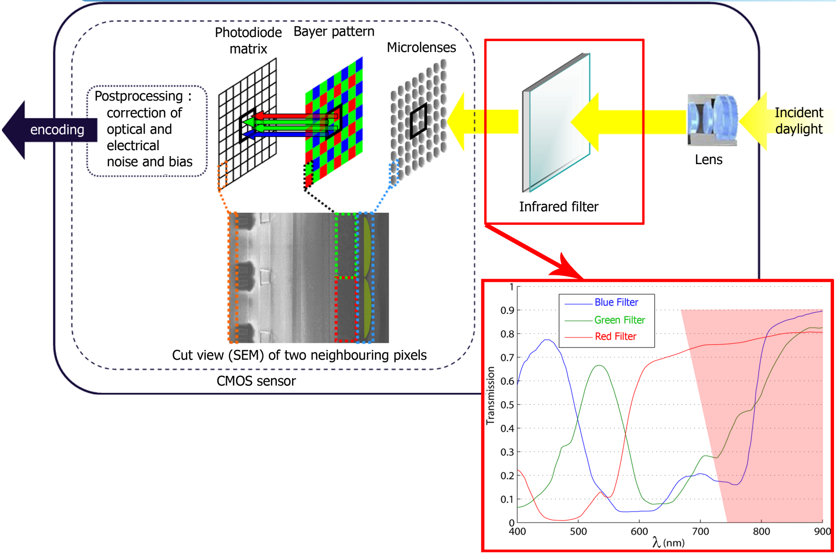

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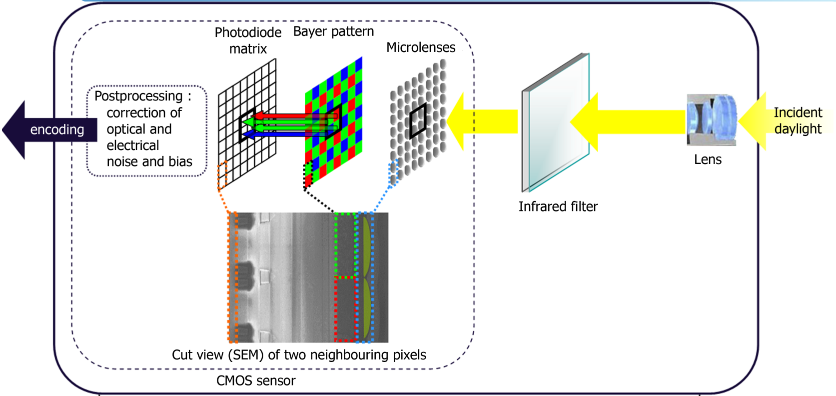

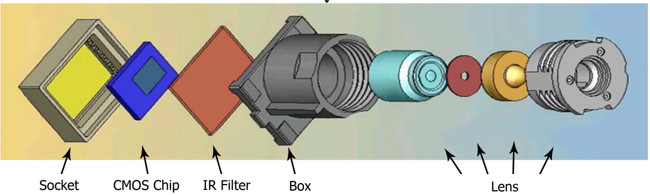

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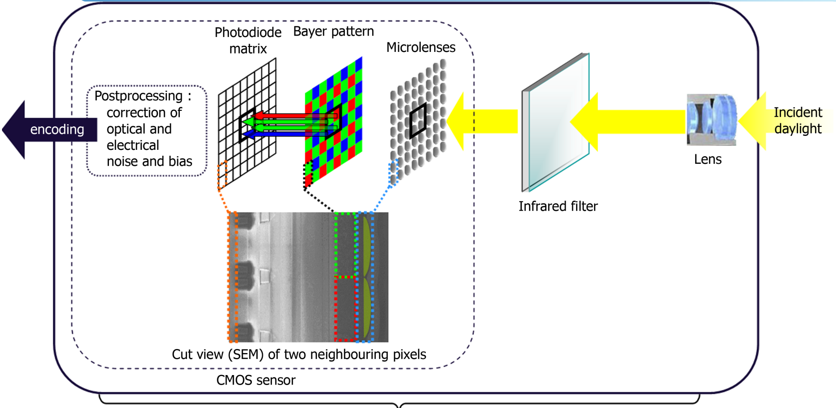

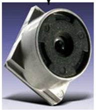

Unavoidable consequences of the miniaturization

CMOS imagers: A complex diffraction gratings.

OPTICAL and technological issues

- Reduction of pixel pitch = transverse dimensions.
- Unchanged thickness:

Modeling issues

- Rising of diffraction

Unavoidable consequences of the miniaturization

CMOS imagers: A complex diffraction gratings.

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- Reduction of pixel pitch = transverse dimensions.
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  - absorbing color resins $\approx$ 50% of the stack thickness
  - Transmission spectral profile set by thickness

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USING METALLIC CROSSED-GRATINGS

Modeling issues

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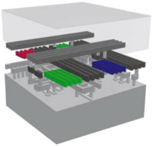

Example[1]

Modeling issues

- Rising of diffraction

[1] Catrysse *et al.*, J. Opt. Soc. Am. A **20**(12), 2003

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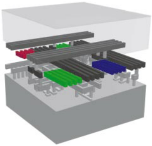
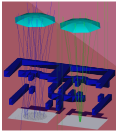


Example[1]

Modeling issues

- Rising of diffraction
  - Snell-Descartes lows no longer valid
  - 

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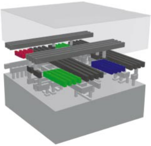

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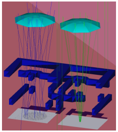
  - 

SOLVING MAXWELL EQUATIONS

[1] Catrysse *et al.*, J. Opt. Soc. Am. A **20**(12), 2003

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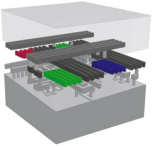

Example[1]

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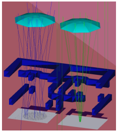
  - 

SOLVING MAXWELL EQUATIONS

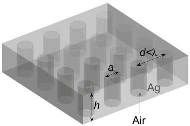
Requires a very flexible and general method.

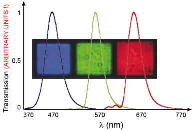
[1] Catrysse *et al.*, J. Opt. Soc. Am. A **20**(12), 2003

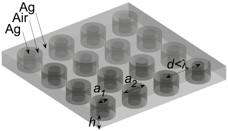
Subwavelength filtering in the visible range.

Independently from CMOS imagers domain

Some bibliographical leads on 0^{th}-order diffractive spectral filtering independent from incident polarization, angle, in the visible frequency range:

- Crossed-gratings made of cylindrical holes [1] in a thin silver layer
- 

- 

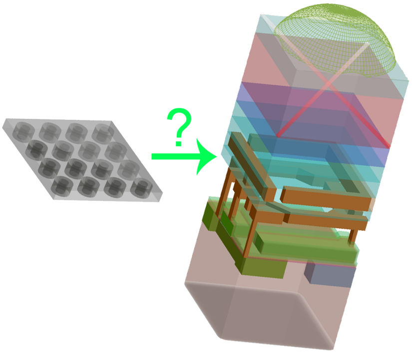
- Crossed-gratings made of cylindrical annular apertures [2] in a thin silver layer
- 

- Bandpass spectral profile with a 90% transmission at $\lambda = 700\,\text{nm}$

[1] Barnes *et al.*, Nature, **424**, 2003

[2] Poujet *et al.*, Opt. Lett., **32**(20), 2007

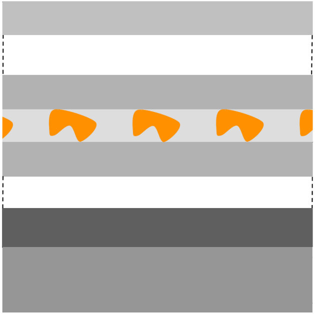
Application to a CMOS pixel



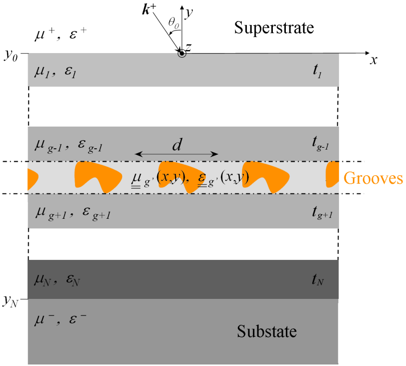
- Looking for resonant phenomena inside a 3D complex structure
- Requires a model able to represent closely both the geometry and constitutive materials of the pixel pixel
- A 2D step is essential.

Part 2

A diffracted field formulation of the FEM



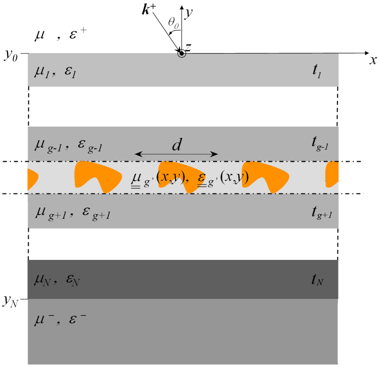
A diffracted field formulation of the FEM

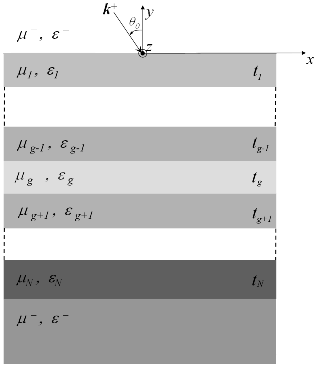


Find the unique $(\mathbf{E},\mathbf{H})$ solution of:

$$\begin{cases} \text{curl}\,\mathbf{E} = i\,\omega\,\mu_0\,\mu\,\mathbf{H} \\ \text{curl}\,\mathbf{H} = -i\,\omega\,\varepsilon_0\,\varepsilon\,\mathbf{E} \end{cases}$$

A diffracted field formulation of the FEM





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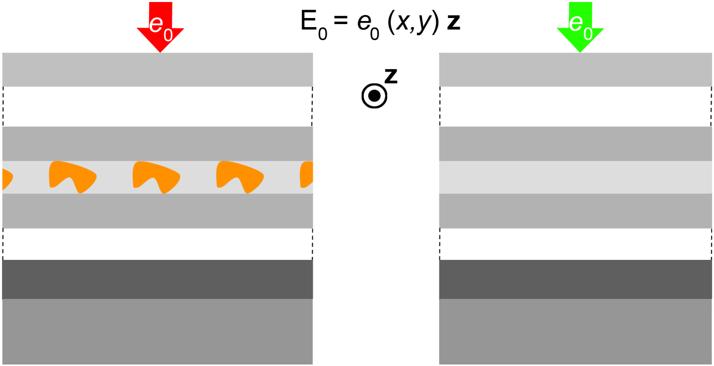

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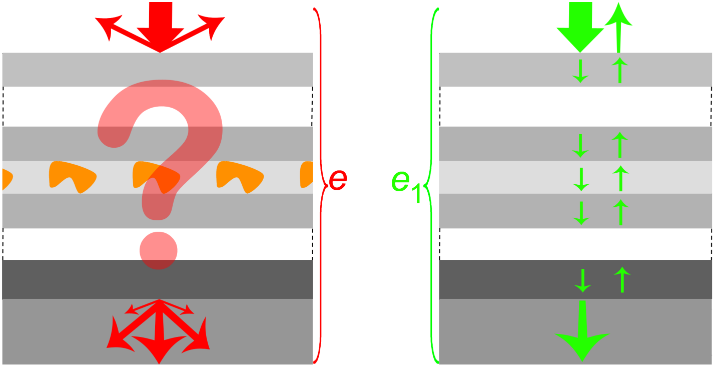
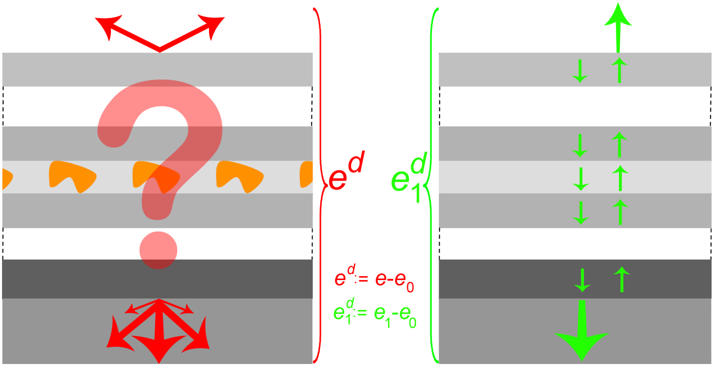


$\mathrm{div}\big(\mu(x,y)\,\mathrm{grad}\,e\big) + k_0^2\,\varepsilon(x,y)\,e = 0$

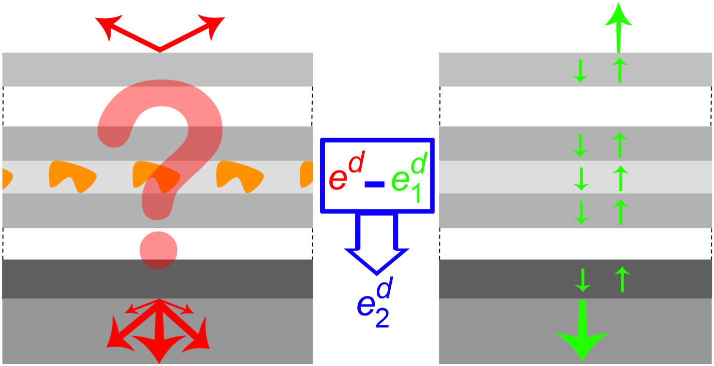
$\mathrm{div}\big(\mu_1(y)\,\mathrm{grad}\,e_1\big) + k_0^2\,\varepsilon_1(y)\,e_1 = 0$

A diffracted field formulation of the FEM



$$\mathrm{div}(\mu(x,y)\,\mathrm{grad}\, e^d) + k_0^2\, \varepsilon(x,y)\, e^d = 0 \quad (1) \qquad \mathrm{div}(\mu_1(y)\,\mathrm{grad}\, e_1^d) + k_0^2\, \varepsilon_1(y)\, e_1^d = 0 \quad (2)$$

A diffracted field formulation of the FEM

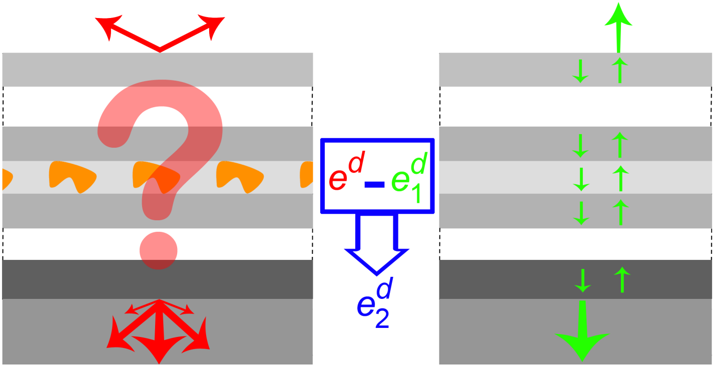


$$\mathrm{div}(\mu(x,y)\,\mathrm{grad}\,e^d) + k_0^2\,\varepsilon(x,y)\,e^d = 0 \quad (1)$$

$$\mathrm{div}(\mu_1(y)\,\mathrm{grad}\,e_1^d) + k_0^2\,\varepsilon_1(y)\,e_1^d = 0 \quad (2)$$

$$e_2^d := e - e_1 = e^d - e_1^d \quad (3)$$

A diffracted field formulation of the FEM



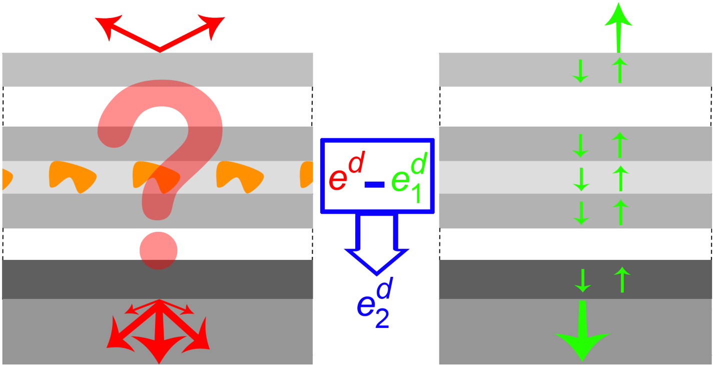
$$\mathrm{div}(\mu(x,y)\,\mathrm{grad}\,e^d) + k_0^2\,\varepsilon(x,y)\,e^d = 0 \quad (1)$$

$$\mathrm{div}(\mu_1(y)\,\mathrm{grad}\,e^d_1) + k_0^2\,\varepsilon_1(y)\,e^d_1 = 0 \quad (2)$$

$$e^d_2 := e - e_1 = e^d - e^d_1 \quad (3)$$

$$(1)-(2) \Longrightarrow \mathrm{div}(\mu(x,y)\,\mathrm{grad}\,e^d_2) + k_0^2\,\varepsilon(x,y)\,e^d_2 = \boxed{\text{S(x,y)}}$$

A diffracted field formulation of the FEM



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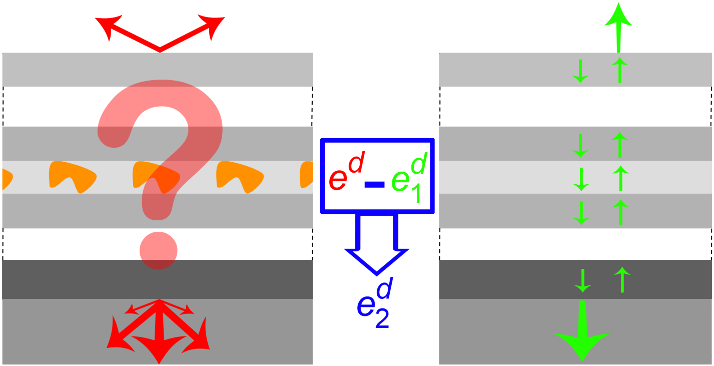
$\boxed{\text{S}}$ is localized in  and only depends on:

A diffracted field formulation of the FEM



$$\mathrm{div}(\mu(x,y)\,\mathrm{grad}\,e_2^d) + k_0^2\,\varepsilon(x,y)\,e_2^d = \boxed{\text{S(x,y)}}$$

$\boxed{\text{S}}$ is localized in  and only depends on:

- e_1^d (known)

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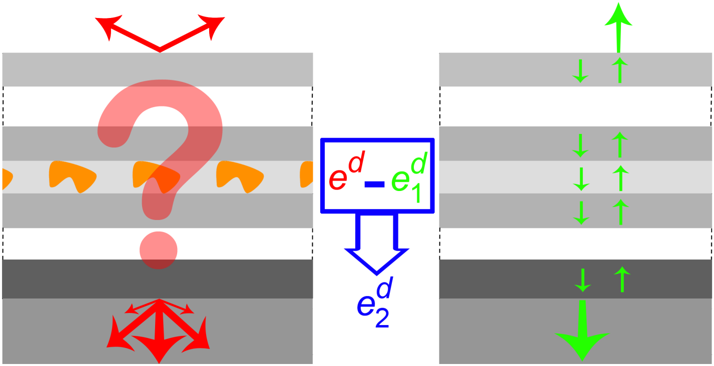
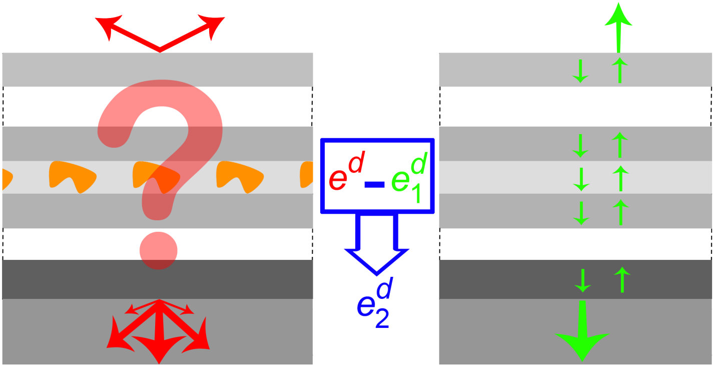


$$\mathrm{div}(\mu(x,y)\,\mathrm{grad}\, e^d_2) + k_0^2\, \varepsilon(x,y)\, e^d_2 = \boxed{\mathrm{S(x,y)}}$$

$\boxed{\mathrm{S}}$ is localized in the diffractive element and only depends on:

- e^d_1 (known)
- Properties of the diffractive element.

A diffracted field formulation of the FEM

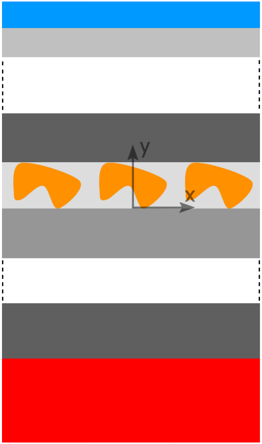


$$\text{div}(\mu(x,y)\,\text{grad}\,e_2^d) + k_0^2\,\varepsilon(x,y)\,e_2^d = \boxed{\text{S(x,y)}}$$

S is localized in [diffractive element] and only depends on:

- e_1^d (known)
- Properties of the diffractive element.
- Properties of the g^{th} layer.

Sum up of the principle of *removal of "infinite" issues*[1] – Computational Domain



[1] Demésy *et al.*, Optics Express **15**, 2007

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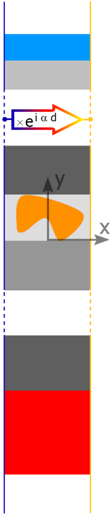
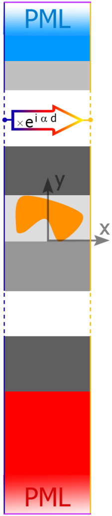


- Infinity of periods + quasi-periodicity of the incident plane wave
  ⟶ Quasi-periodic (or Bloch) conditions

[1] Demésy *et al.*, Optics Express **15**, 2007

Sum up of the principle of *removal of "infinite" issues*[1] – Computational Domain



- Infinity of periods + quasi-periodicity of the incident plane wave
  - ⟶ Quasi-periodic (or Bloch) conditions
- Infinite Substrate/Superstrate
  - ⟶ Rectangular PML

[1] Demésy *et al.*, Optics Express **15**, 2007

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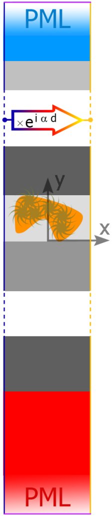
Sum up of the principle of *removal of "infinite" issues*[1] – Computational Domain



- Infinity of periods + quasi-periodicity of the incident plane wave
  ⟶ Quasi-periodic (or Bloch) conditions
- Infinite Substrate/Superstrate
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  ⟶ Equivalence to a radiative problem with localized sources

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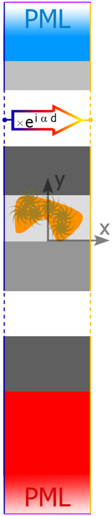
Sum up of the principle of *removal of "infinite" issues*[1] – Computational Domain



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- Plane wave sources at infinity
  ⟶ Equivalence to a radiative problem with localized sources
- Meshing of the structure: 2^{nd} order nodal elements
- Weak form associated to equation (3)
- Solving thanks to a direct solver adapted to sparse matrix (MUMPS)

[1] Demésy *et al.*, Optics Express **15**, 2007

Weak formulation and discrete problem

- For example, let us consider nodal elements (first order = circus big top) built on a triangular mesh m with its set of nodes is denoted N.
- Projection of the field e_2^d on this (non-orthogonal) basis:

$$e_2^{d,m}(x,y) = \sum_{i \in N} \alpha_i \, \lambda_i(x,y)$$

- "Weak form" of the problem:

$$\mathscr{R}_{\mu,\varepsilon}(e_2^d, e') = -\int_\Omega \left(\mu \nabla e_2^d\right) \cdot \nabla \overline{e'} + k_0^2 \varepsilon \, e_2^d \, \overline{e'} \, d\Omega + \int_{\partial\Omega} \overline{e'} \left(\mu \nabla e_2^d\right) \cdot \mathbf{n}_{|\partial\Omega} \, dS \qquad (1)$$

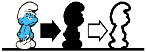
- e_2^d, solution of the radiation problem, is therefore the element of $L^2(\text{curl}, d, k)$ of quasiperiodic functions (i.e. such that $u(x,y) = u_\#(x,y)e^{ikx}$ with $u_\#(x,y) = u_\#(x+d,y)$, a d-periodic function of $H^1(\text{div})$) on Ω such that:

$$\forall e' \in H^1(\text{div}, d, k), \; \mathscr{R}_{\mu,\varepsilon}(e_2^d, e') = -\mathscr{R}_{\mu-\mu_1, \varepsilon-\varepsilon_1}(e_1, e'). \qquad (2)$$

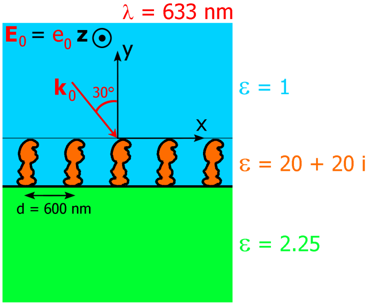
- According to the Galerkin formulation, we choose the set of basis function λ_i as set of weight function e', which leads to a final algebraic system of the form:

$$A \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = b \text{ where } A \text{ is a sparse matrix.}$$

Arbitrary geometry: an example of edge detection

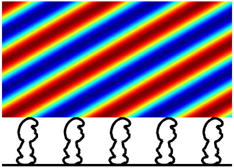


Studied configuration: TM case, oblique incidence, strong losses



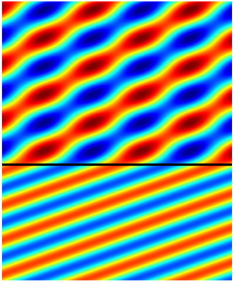
Energetic considerations

Diffraction Efficiencies – Losses – Energy Balance



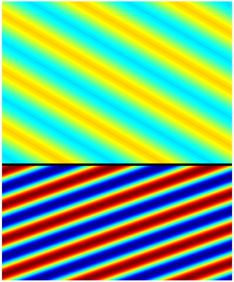
$$e_0(x, y) = \exp(i(\alpha x + \beta^{sup} y))$$

$\Re e[e_0]$, Incident plane wave



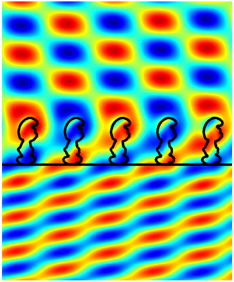
$$e_1(x,y) = e_0(x,y) + \exp(i\alpha x)\begin{cases} r\exp(-i\beta^{sup}y) & \text{for} \quad y > 0 \\ t\exp(i\beta^{sub}y) & \text{for} \quad y < 0 \end{cases}$$

$\Re e[e_1]$, Total field solution of the ancillary problem (plane diopter)



$$e_1^d(x,y) = \exp(i\,\alpha\,x) \begin{cases} r\exp(-i\,\beta^{sup}\,y) & \text{for} \quad y > 0 \\ t\exp(i\,\beta^{sub}\,y) & \text{for} \quad y < 0 \end{cases}$$

$\Re e[e_1^d]$, Field “diffracted” by the plane diopter



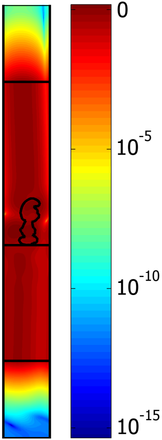
$$S(x,y) = k_0^2\,(\varepsilon_{smurf} - \varepsilon_{air})\ \exp(\alpha\,x + \beta^{sup}\,y) + k_0^2\,(\varepsilon_{smurf} - \varepsilon_{air})\,r\exp(\alpha\,x - \beta^{sup}\,y)$$

$$\mathrm{div}(\mu(x,y)\ \mathrm{grad}\,e_2^d) + k_0^2\,\varepsilon(x,y)\,e_2^d = \boxed{\text{S(x,y)}}$$

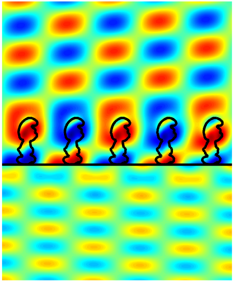
$\boxed{\Re e[e_2^d]}$, FEM-calculated field or radiated field

Energetic considerations

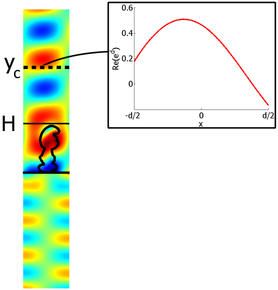
Diffraction Efficiencies – Losses – Energy Balance



$\log(|e_2^d|)$

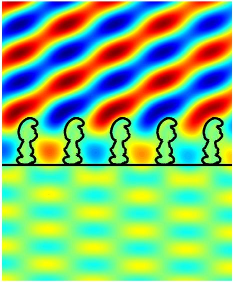


$\Re e[e^d]$, Diffracted field

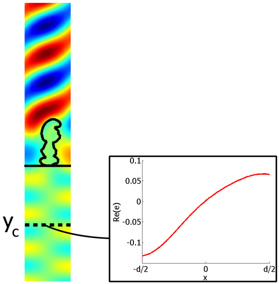


- $R_n := r_n \overline{r_n} \frac{\beta_n^+}{\beta^+}$ for $y_c > H$
- $r_n = \frac{1}{d} \int_{-d/2}^{d/2} e^d(x, y_c)\, e^{-i(\alpha_n x + \beta_n^+ y_c)}\, dx$
- Reflected propagative orders: $R_0 = 0.1138$ and $R_1 = 0.1846$

$\Re e[e^d]$, Diffracted field



$\Re e[e]$, Total field

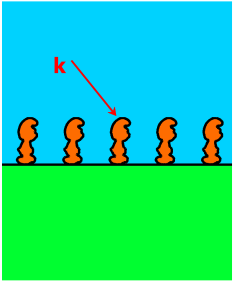


- $T_n := t_n \overline{t_n} \dfrac{\beta_n^-}{\beta^-} \dfrac{\gamma^+}{\gamma^-}$ for $y_c < 0$
- $t_n = \dfrac{1}{d} \displaystyle\int_{-d/2}^{d/2} e(x, y_c)\, e^{-i(\alpha_n x - \beta_n^- y_c)}\, dx$
- Transmitted propagative orders: $T_0 = 0.0704$ and $T_1 = 0.0711$

$\Re e[e]$, Total field

- Losses : $\frac{1}{2}\int_{smurf} \omega\, \varepsilon_0\, \varepsilon'' \left|e(x,y)\right|^2 \mathrm{d}x\, \mathrm{d}y$
- Losses (in fraction of incident energy) : $Q = 0.5601$

$\frac{1}{2}\,\sigma\,|e|^2$



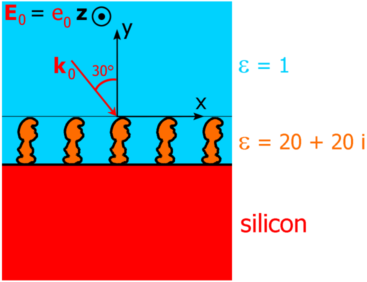
Energy Balance:

$$R + T + Q = (R_0 + R_1) + (T_0 + T_1) + Q = \begin{array}{rr} & 0.1138 \\ + & 0.1846 \\ + & 0.0704 \\ + & 0.0711 \\ + & \underline{0.5601} \\ = & 1.0001 \end{array}$$

“Quantum” efficiency in the case of a semi-conductor substrate

A grating on a n^{+}/p junction

Studied configuration: semi-conductor substrate



Hypothesis

- abrupt junction
- unilateral junction
- conversion efficiency ≈ 1

$$\mathrm{QE} = \frac{\text{number of electrons participating to the photo-current}}{\text{number of incident photons}}$$

“Quantum” efficiency in the case of a semi-conductor substrate

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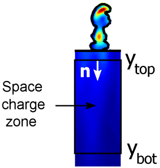
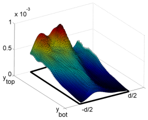




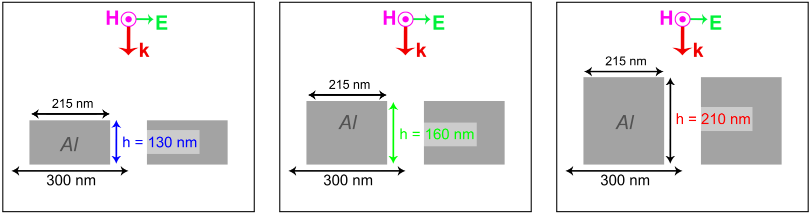
$$\mathrm{QE}(\lambda) = \frac{\displaystyle\int_{-d/2}^{d/2}\int_{y_{\mathrm{bot}}}^{y_{\mathrm{top}}} -\operatorname{div}\left(\mathbf{S}_{\mathrm{moy}}(x,y,\lambda)\right)\mathrm{d}x\,\mathrm{d}y}{\displaystyle\int_{-d/2}^{d/2}\mathbf{n}\cdot\mathbf{S}_0\,\mathrm{d}x}$$

$|\mathbf{S}_{moy}|$, norm of Poynting vector

Demo!

https://gitlab.onelab.info/doc/models/tree/master/DiffractionGratings/

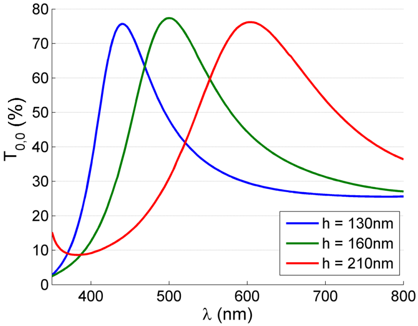
Aluminum color filters



Let us map $|H_z|^2(x, y, \lambda)$ vs $T_{0,0}(\lambda)$

Aluminum color filters

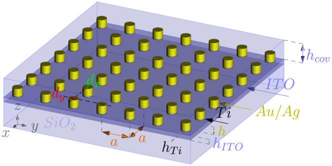
Aluminum color filters



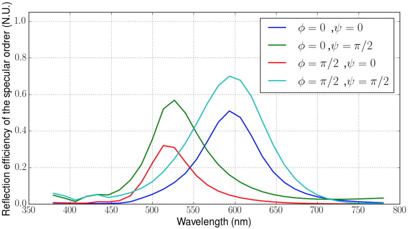
Later...

Frequency selective reflective surface with silver nano-particles [1]

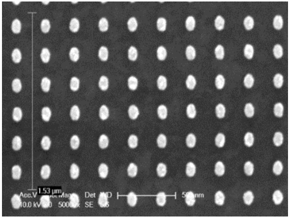
Structure



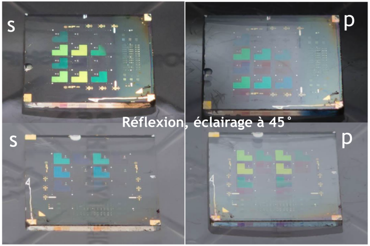
Design - “optimization”



Fab



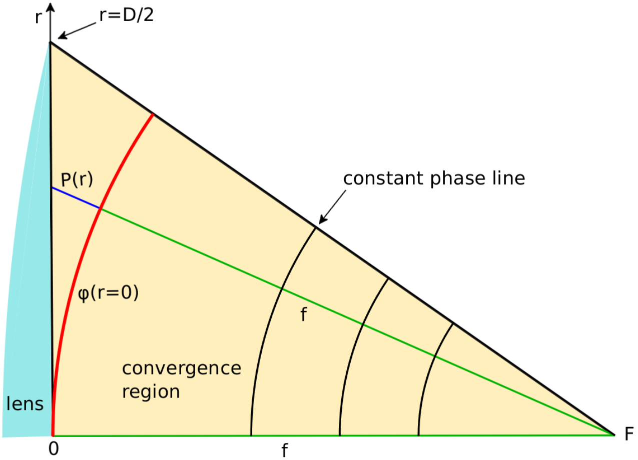
Carac



[1] Y. Brûlé, G. Demésy, A.-L. Fehrembach, B. Gralak, E. Popov, G. Tayeb, M. Grangier, D. Barat, H. Bertin, P. Gogol, and B. Dagens, “Design of metallic nanoparticle gratings for filtering properties in the visible spectrum”, Appl. Opt. 54, 010359 (2015).

"Metasurfaces"...

Sub-wavelength phase control [1]



"Metasurfaces"...

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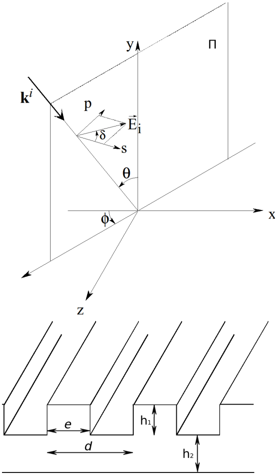

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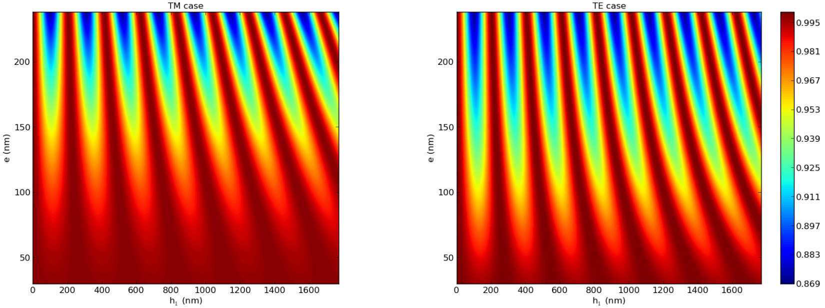

“Metasurfaces”. . .

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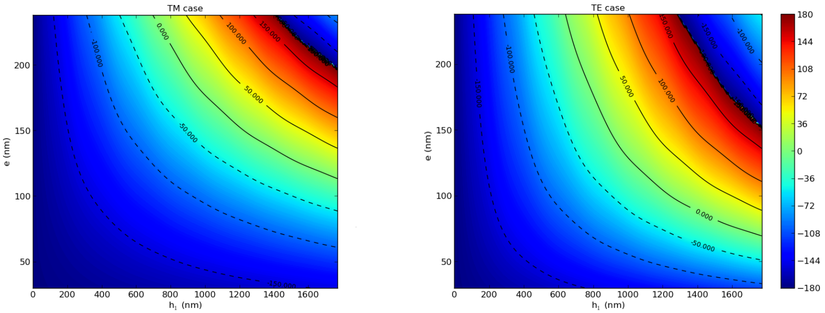

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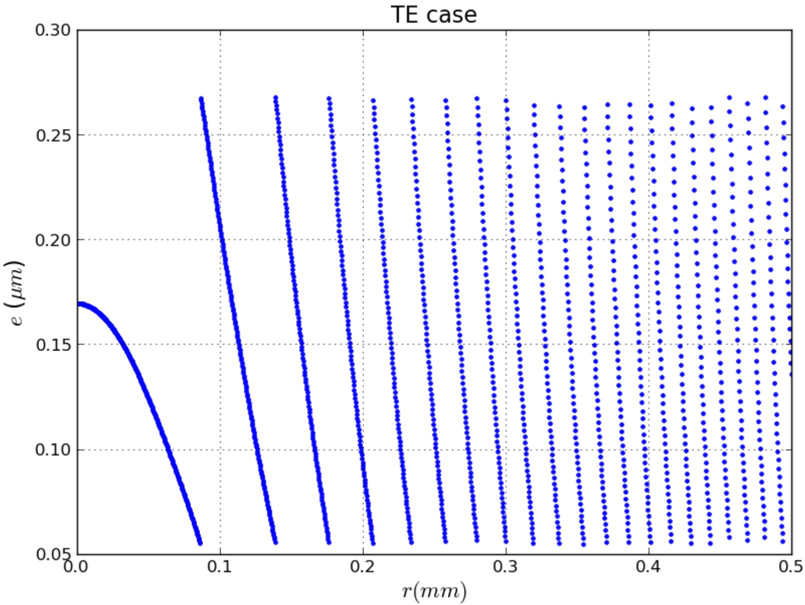

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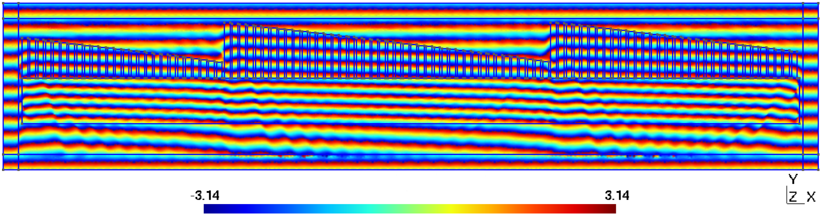

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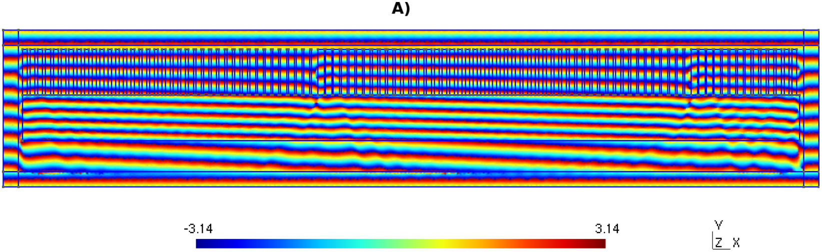
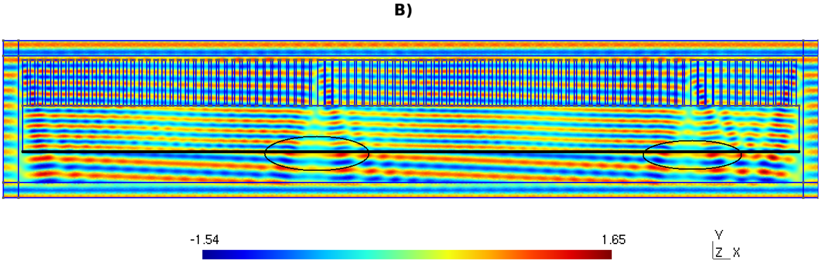




Conclusion

Solving Maxwell's equations in small but open boxes.

Applications for filtering.

. . . We are also calculating mode. . .

The eigenvalue problem

2D p-polarization case: $\mathbf{H} = H_z(x,y)\mathbf{z}$ and $\mathbf{E} = \mathbf{E}(x,y) = E_x(x,y)\mathbf{x} + E_y(x,y)\mathbf{y}$

We are looking for non trivial solution of the source-free Helmholtz equation:

- $\mathscr{L}_e^{3D}(\mathbf{E}) := \underline{\underline{\varepsilon_r}}(\mathbf{x}, \omega)^{-1}\,\mathbf{curl}\left(\underline{\underline{\mu_r}}^{-1}\,\mathbf{curl}\,\mathbf{E}\right) = \left(\frac{\omega}{c}\right)^2 \mathbf{E}$
- *i.e.* the eigenvalues ω_n and associated eigenvectors $\mathbf{E}_n$ of the operator $\mathscr{L}_e^{3D}$
- $\mathscr{L}_e^{3D}(\mathbf{E})$ depends of ω we are looking for!

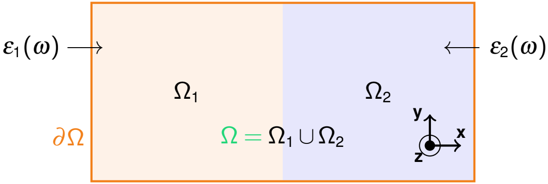
Two possible solutions:

- Physical linearization: construction of an augmented system where auxiliary fields are added to $(\mathbf{E}, \mathbf{H})$ [1]
- Numerical linearization [2]

[1] Y. Brûlé, B. Gralak, and G. Demésy, "Calculation and analysis of the complex band structure of dispersive and dissipative two-dimensional photonic crystals", J. Opt. Soc. Am. B **33**, 691-702 (2016)

[2] J. E. Roman, C. Campos, E. Romero and A. Tomas. SLEPc Users Manual. Tech. Rep. DSIC-II/24/02 - Revision 3.7, Universitat Politècnica de València, 2016.

Toy example: the “bi-Drudy Bi-box”



Dispersion relation: Semi-analytical transcendental equation

- s-polarization: $\dfrac{1}{\beta_1(\omega_n)}\tan[\beta_1(\omega_n)a] + \dfrac{1}{\beta_2(\omega_n)}\tan[\beta_2(\omega_n)a] = 0$
- p-polarization: $\dfrac{\beta_1(\omega_n)}{\varepsilon_1(\omega_n)}\tan[\beta_1(\omega_n)a] + \dfrac{\beta_2(\omega_n)}{\varepsilon_2(\omega_n)}\tan[\beta_2(\omega_n)a] = 0$

With β_1 and β_2 two complex functions of ω :

$$\beta_j(\omega_n) = \sqrt{\frac{\omega_n^2}{c^2}\varepsilon_j(\omega_n) - \frac{q^2\pi^2}{a^2}} \quad \text{with} \quad j \in \{1,2\}, \tag{3}$$

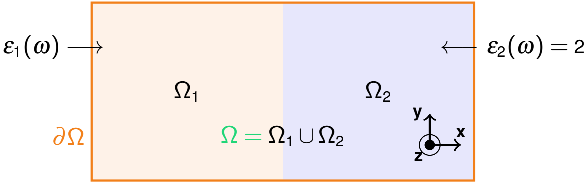
where $q \in \mathbb{N}^\star$ for s-polarization and $q \in \mathbb{N}$ for p-polarization.

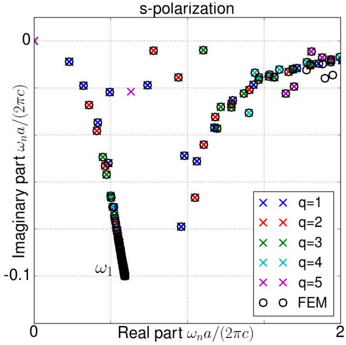
Toy example: the “bi-Drudy Bi-box”

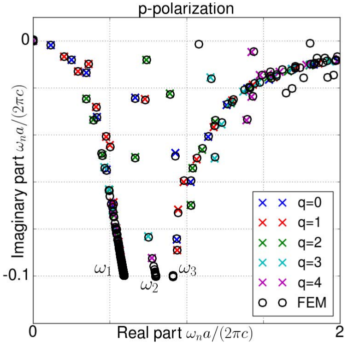
$$\varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + \gamma i \omega - \omega_0^2} =$$

with $\varepsilon_\infty = 3.0\,,\quad \dfrac{\omega_p a}{2\pi c} = 1.2$

and $\dfrac{\gamma a}{2\pi c} = 0.2\,,\quad \dfrac{\omega_0 a}{2\pi c} = 0.6$







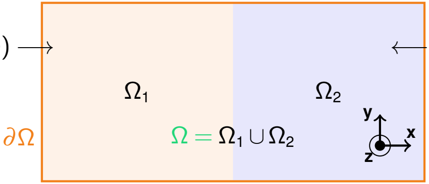
Toy example: the “bi-Drudy Bi-box”

$$\varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + \gamma i \omega - \omega_0^2} = \varepsilon_1(\omega) \longrightarrow$$

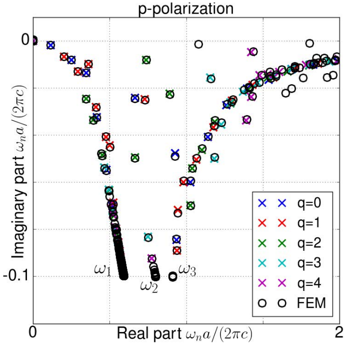
$$\longleftarrow \varepsilon_2(\omega) = 2$$

with $\varepsilon_\infty = 3.0\,, \quad \frac{\omega_p a}{2\pi c} = 1.2$

and $\frac{\gamma a}{2\pi c} = 0.2\,, \quad \frac{\omega_0 a}{2\pi c} = 0.6$



- $\omega_1 : |\varepsilon_1(\omega)| \to \infty$
- $\omega_2 : \varepsilon_1(\omega_2) = -\varepsilon_2$ plasmons
- $\omega_3 : \varepsilon_1(\omega_3) = 0$ spurious
- high frequencies: $\varepsilon_1(\omega) \to \varepsilon_\infty$



p-polarization modes:

Toy example: the “bi-Drudy Bi-box”

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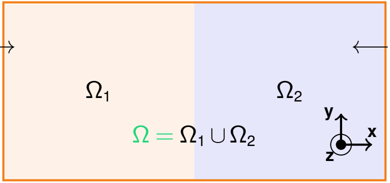
$\longleftarrow \varepsilon_2(\omega) = 2$

$\partial\Omega$



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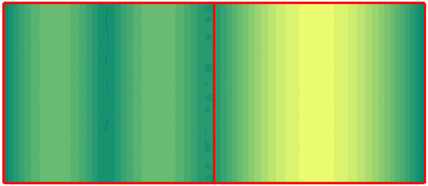

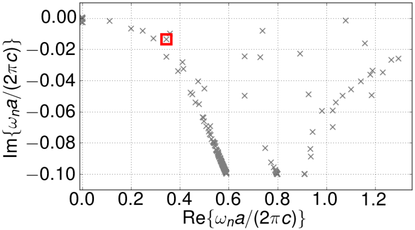

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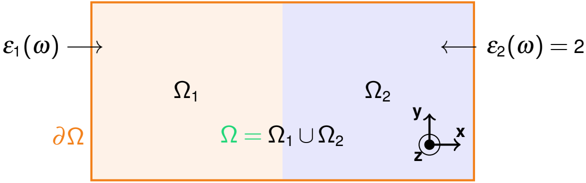

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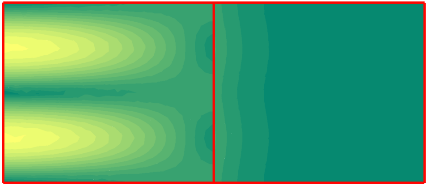

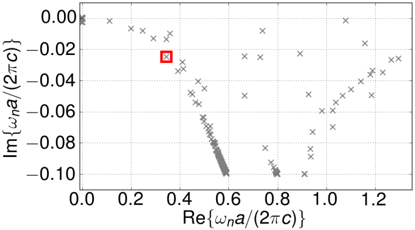

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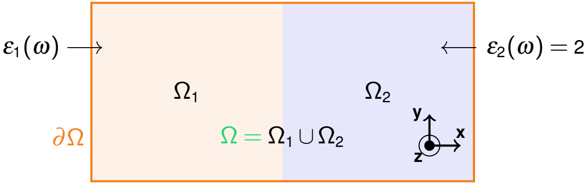

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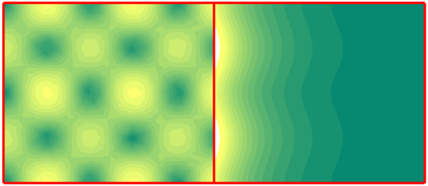

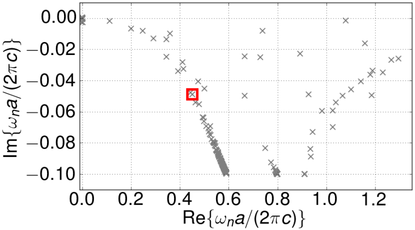

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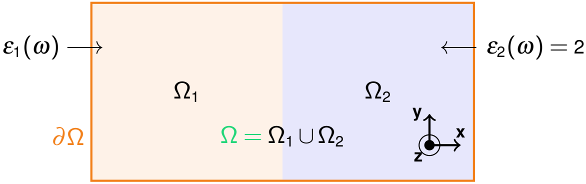

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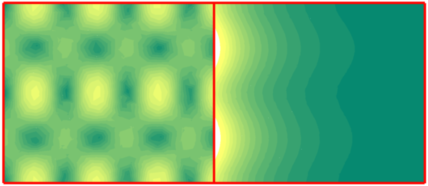

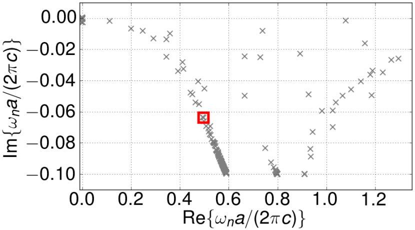

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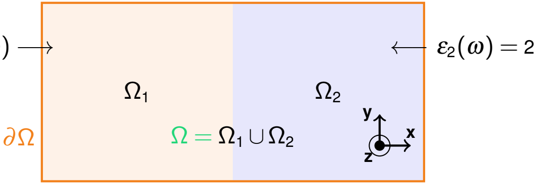

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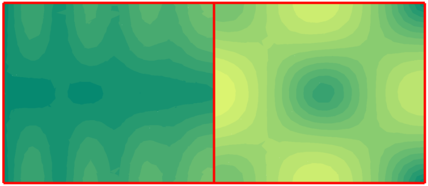

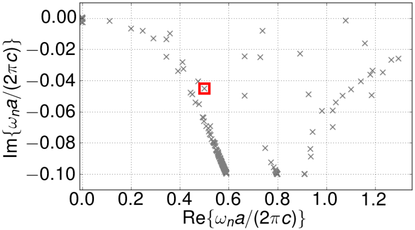

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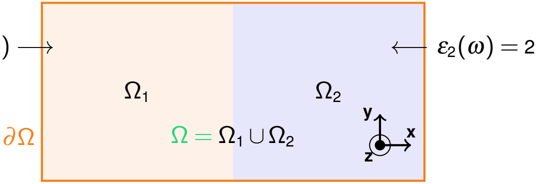

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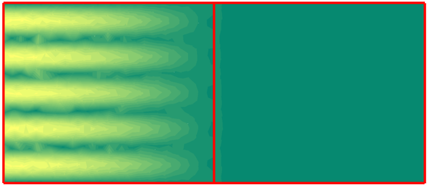

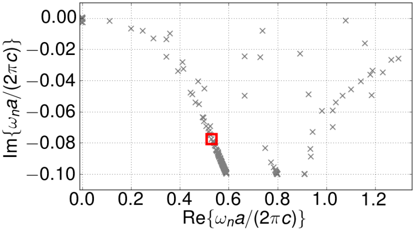

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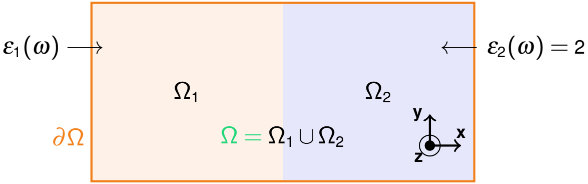

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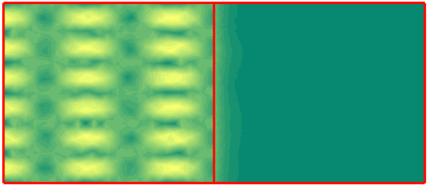

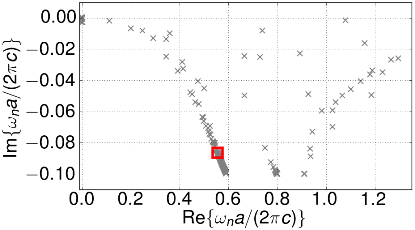

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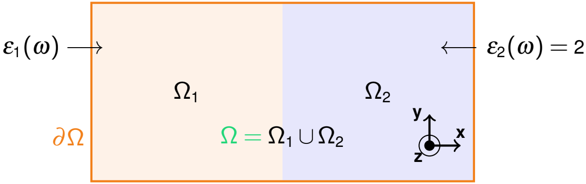

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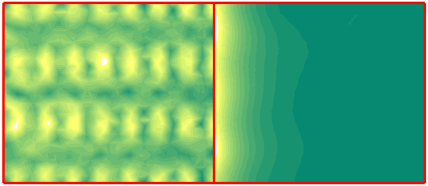

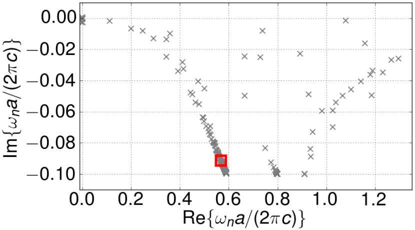

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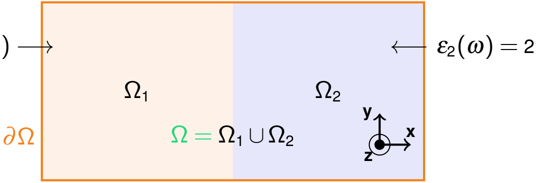

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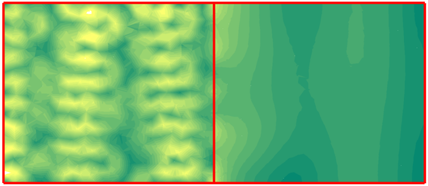

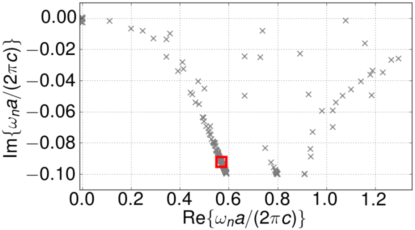

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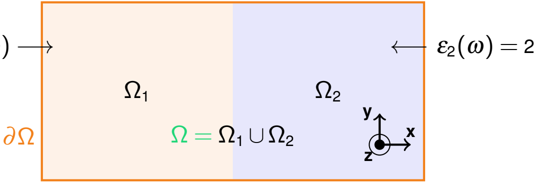

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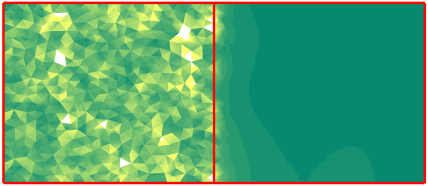

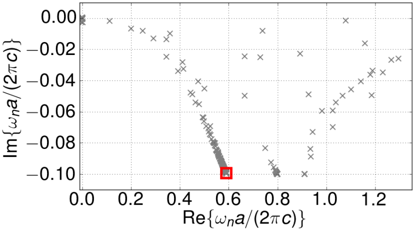

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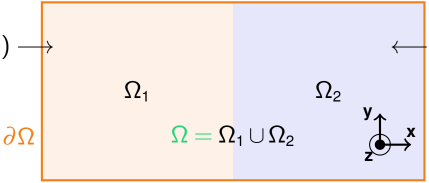

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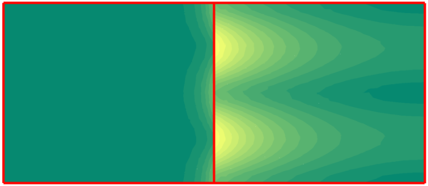

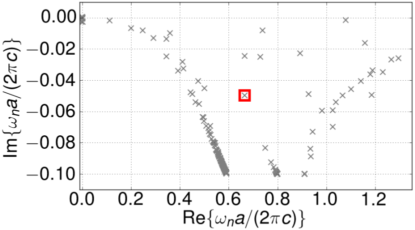

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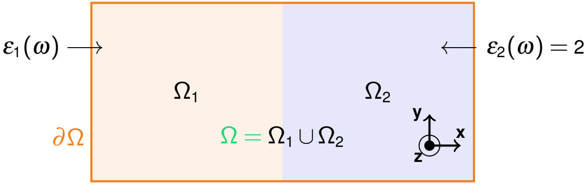

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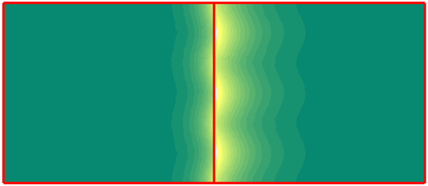

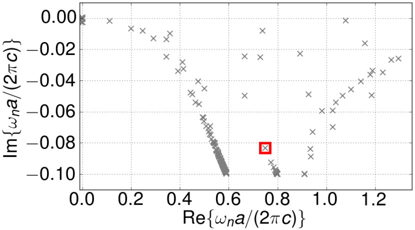

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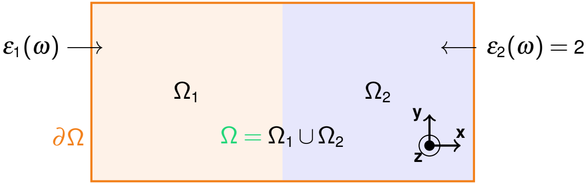

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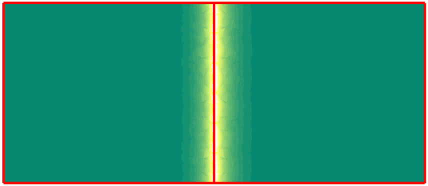

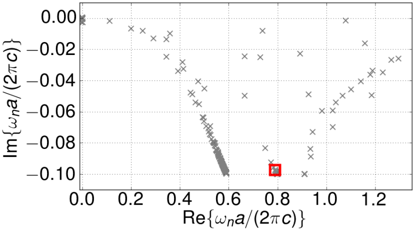

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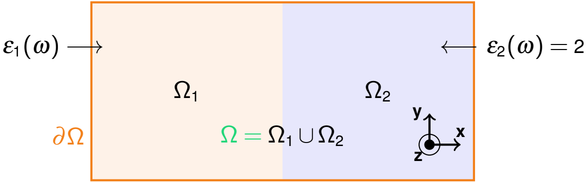

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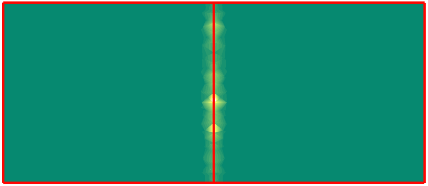

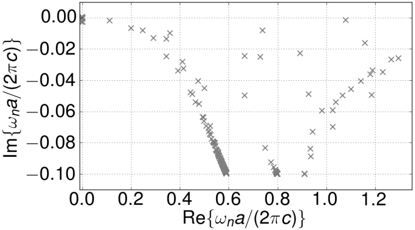

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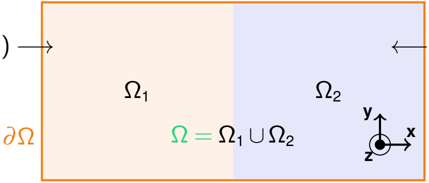

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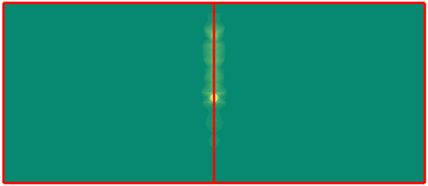

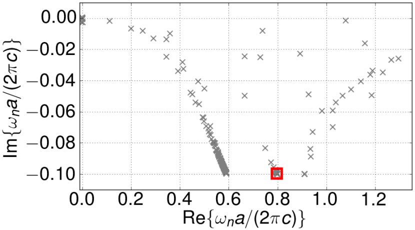

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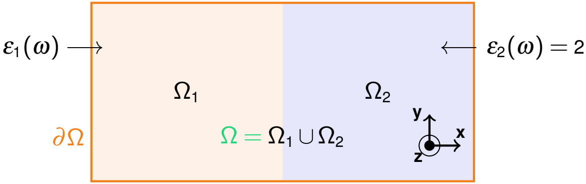

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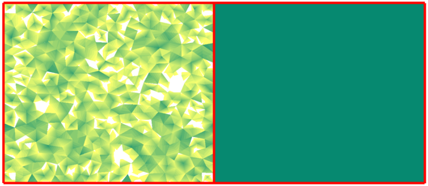

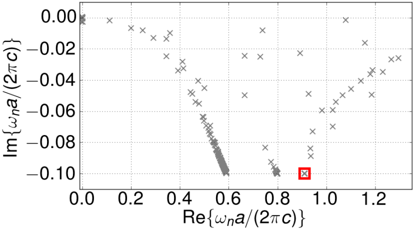

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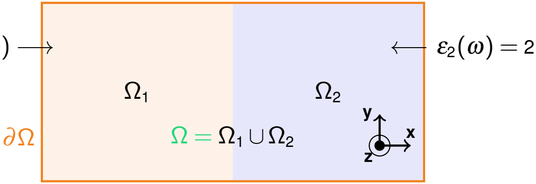

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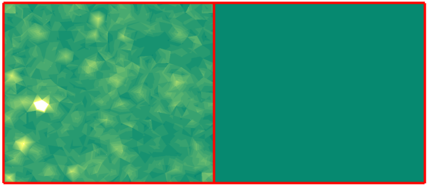

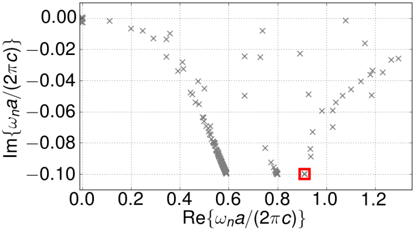

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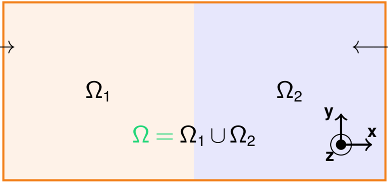
$\partial\Omega$



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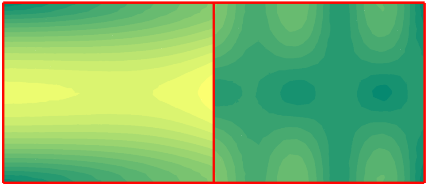

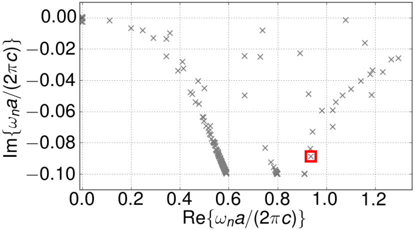

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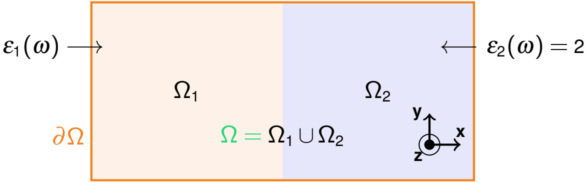

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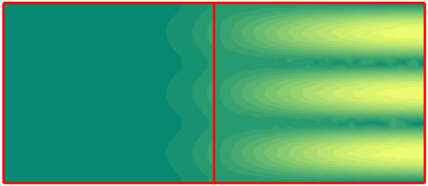

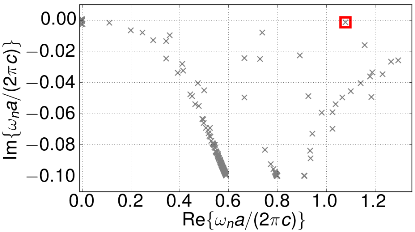

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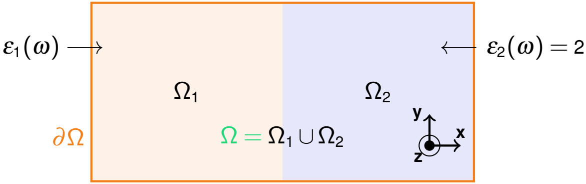
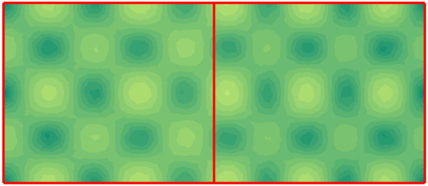
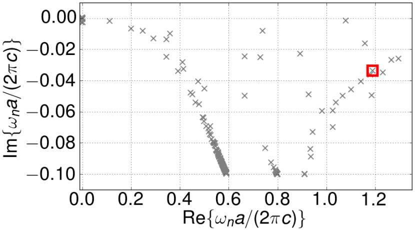


p-polarization modes:

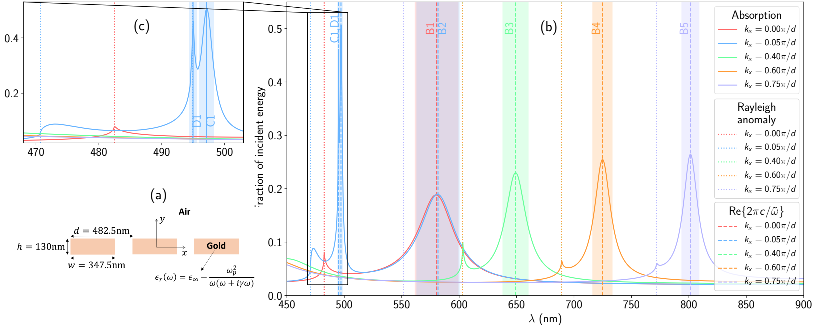




Resonances in frequency dispersive media (ANR RESONANCE)

with F. Zolla, A. Nicolet and B. Gralak (PI: P. Lalanne, LP2N)

Dispersion relation of a grating made of a Drude metal: $\varepsilon_r(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega+i\gamma)}$



G. Demésy *et al.* `https://arxiv.org/abs/1802.02363v1`

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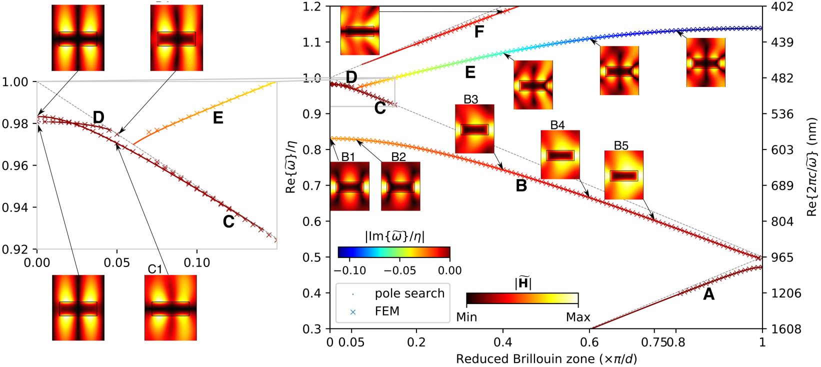
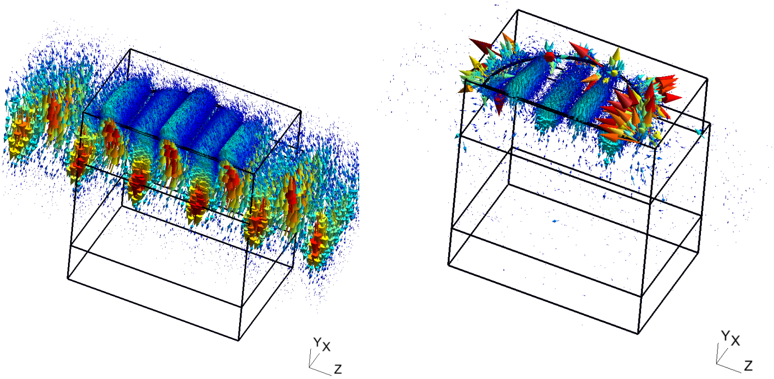


G. Demésy *et al.* `https://arxiv.org/abs/1802.02363v1`

Structured waveguides (ANR LOUISE)

with G. Renversez (PI: V. Nazabal, Institut des Sciences Chimiques de Rennes)

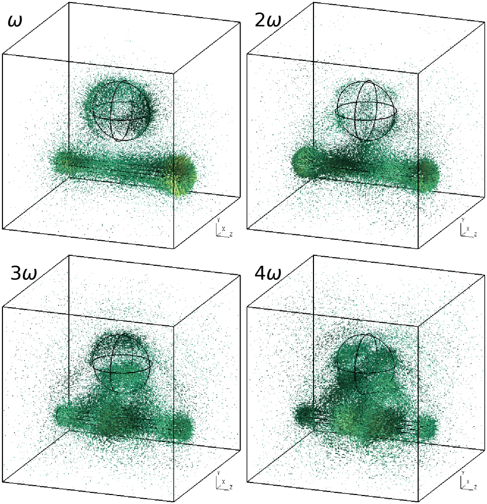
A leaky mode. . . as an incident field for a scattering problem.



Oscillating particle

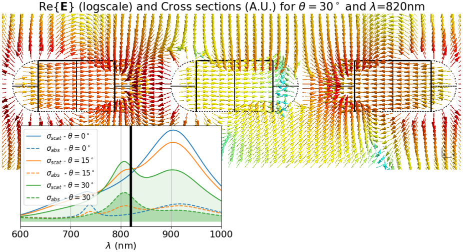
Mauricio Garcia-Vergara's PhD

Oscillating charge: A multiharmonic problem



Other examples (scattering)

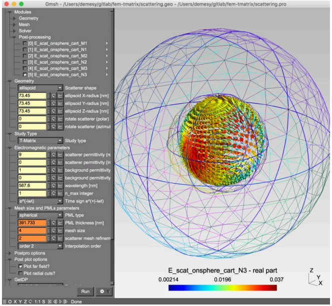
With N. Bonod + UTT



A trimer in a photoresist.

`ACS Photonics, 2018, 5 (3), pp 918-928`

With B. Stout

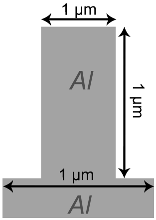


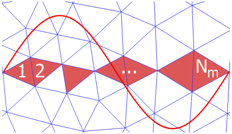
T matrix of an arbitrary scatterer. GNU Model.

`https://arxiv.org/abs/1802.00596v2`

Metallic grating academical case

Comparison to the results of a independent modal method (FMM)





N_M	R_0^{TM}	R_0^{TE}
4	0.7336765	0.8532342
6	0.7371302	0.8456592
8	0.7347466	0.8482817
10	0.7333739	0.8500710
12	0.7346569	0.8494844
14	0.7341944	0.8483238
16	0.7342714	0.8484774
Result given by Granet *et al.*[1]	0.7342789	0.8484781

[1] G. Granet, J. Opt. Soc. Am. A, **16**(10), 1999.

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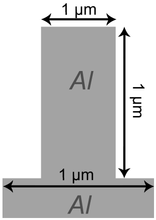

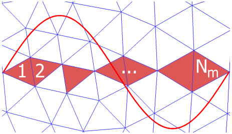

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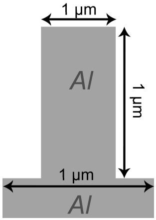

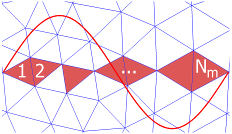

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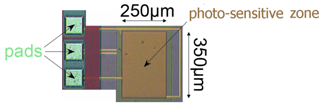
Part 3

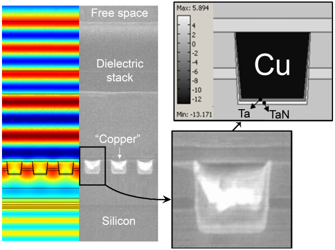
Test Structures presentation

Top view

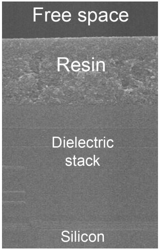


Cross-sectional view

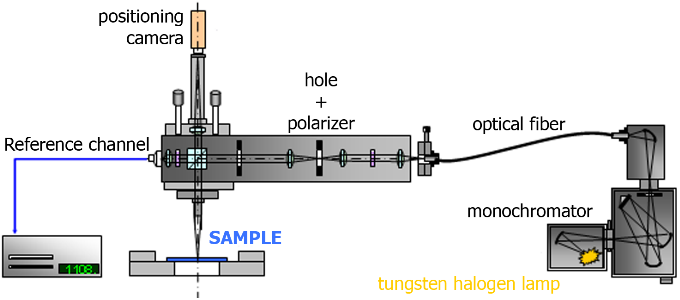
Photodiode with embedded grating



Same photodiode, without any grating

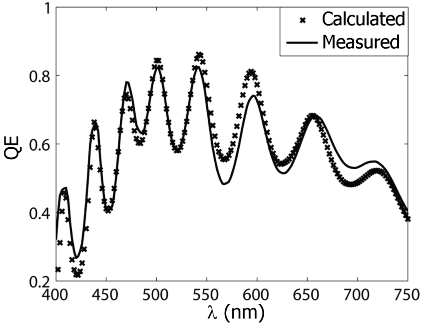


Optical measurement bench



INSTITUT FRESNEL

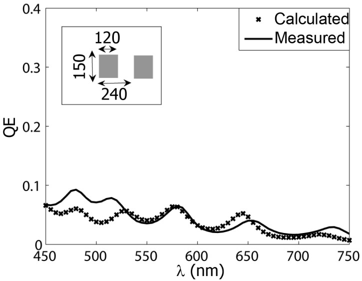
Photodiode toped with a dielectric multilayered stack



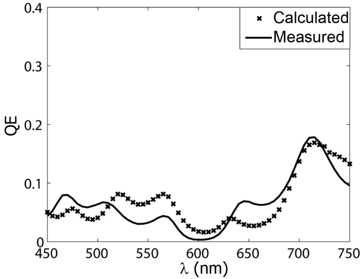
- Very good agreement without any adjustment parameter, provided the precise knowing of:
  - The thickness of each layer (SEM views of cross-sections)
  - The dispersion of each material (ellipsometric measurements)
- Validates both:
  - the use of the measured $\varepsilon(\lambda)$, on which is based the ancillary problem,
  - the validity of the approximation of QE calculation.

Photodiode with an embedded copper grating[1]

TM CASE

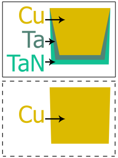


TE CASE

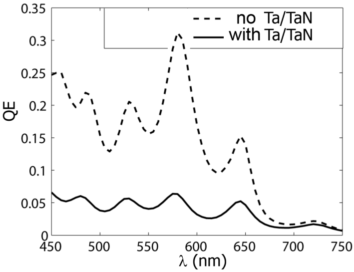


[1] Demésy *et al.*, Optical Engineering **48**, p.058002 (may 2009)

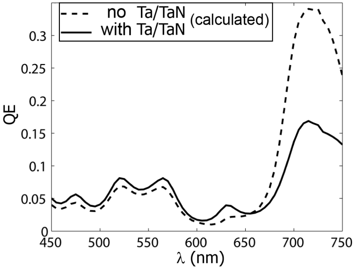
Photodiode with an embedded copper grating[1]



TM CASE

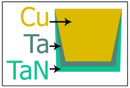


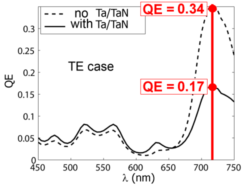
TE CASE

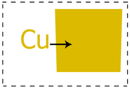


[1] Demésy *et al.*, Optical Engineering **48**, p.058002 (may 2009)

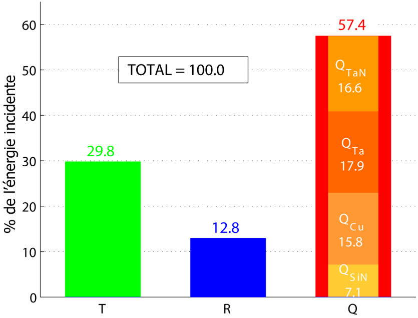
Energy balance – TE case – $\lambda = 720\,\text{nm}$







With Ta/TaN barrier



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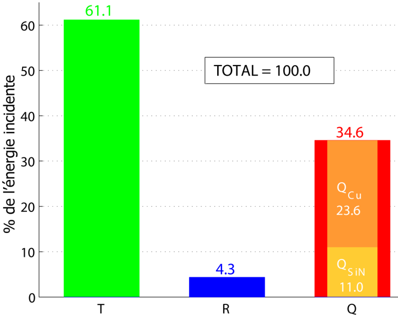
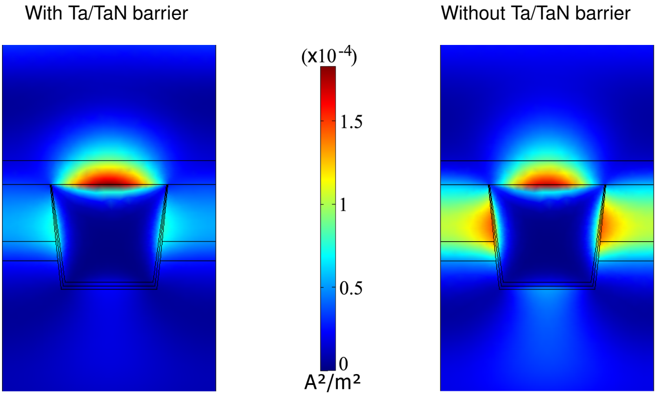


Field maps – TE case – $\lambda = 720\,\mathrm{nm}$



$|H_z|^2$