Reconstructors for Fourier-based wavefront sensors derived from mathematical models

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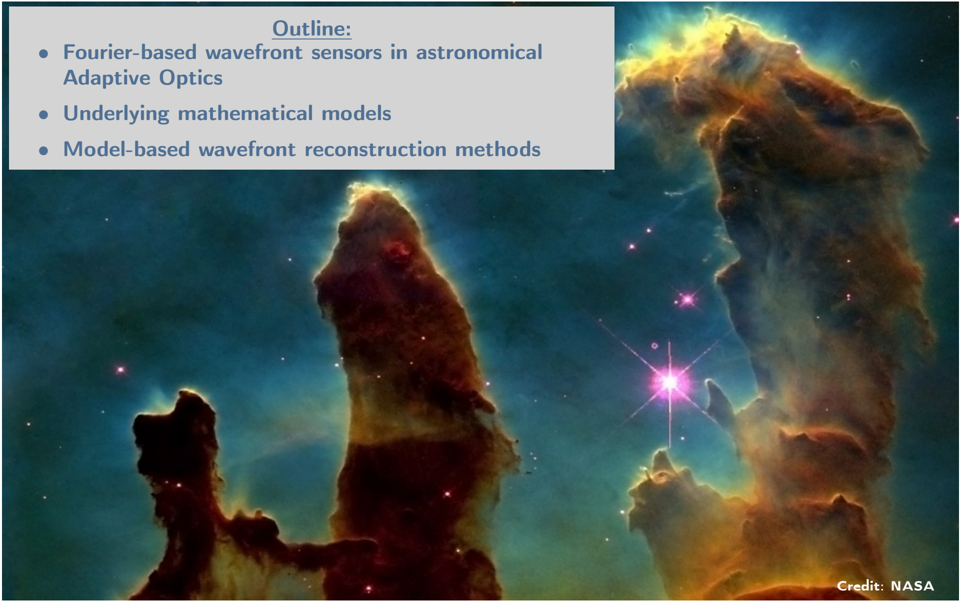
industrial mathematics institute

SFB Tomography Across the Scales

LAM Marseille, February 28, 2019

Outline:

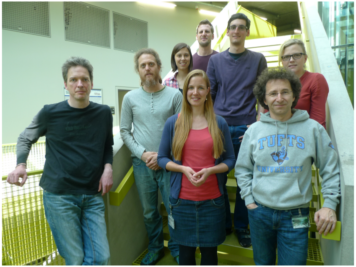
- Fourier-based wavefront sensors in astronomical Adaptive Optics
- Underlying mathematical models
- Model-based wavefront reconstruction methods



Credit: NASA

ELT - the world's biggest eye on the sky

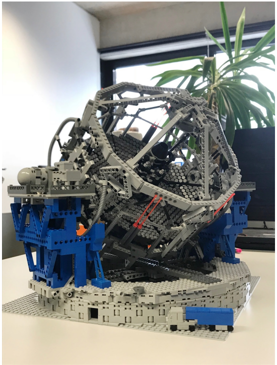
- Austrian scientific contribution:
  "Mathematical algorithms and software for ELT adaptive optics"
- Austrian Adaptive Optics (AAO) team in Linz
- ELT instruments METIS & MICADO



Credit: ESO

Austrian Adaptive Optics

working on:

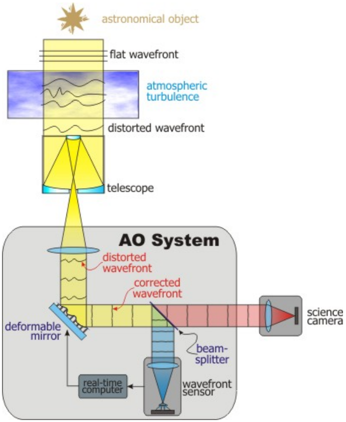


- wavefront reconstruction for ELTs
- Shack-Hartmann and pyramid WFS
- atmospheric tomography
- PSF reconstruction
- optimal control
- ...

https://www.facebook.com/TomographyAcrossTheScales/

Adaptive Optics (AO)

Adaptive Optics is a technique for correcting optical distortions arising during the imaging process ∼ hardware based real-time deblurring



Credit: C. Max

components of AO system:

- deformable mirrors
- wavefront sensors
- control system
  → **inverse problem**

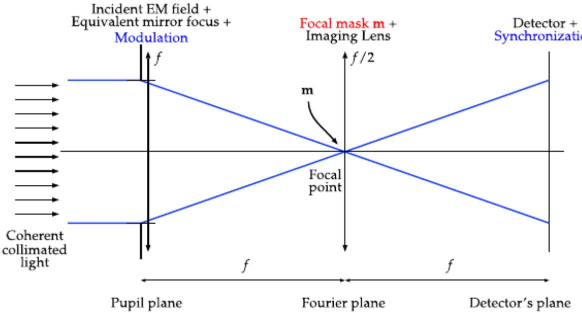
Inverse problem of wavefront reconstruction

Wavefront sensors (WFSs) provide intensity measurements which are related in a (non-linear) way to the wavefront of the incoming light.

$$\underbrace{\mathbf{s}}_{\substack{\text{WFS}\\\text{measurements}}} = \overbrace{\boldsymbol{W}}^{\substack{\text{WFS}\\\text{operator}}} \underbrace{\Phi}_{\substack{\text{incoming}\\\text{phase}}} + \overbrace{\eta}^{\substack{\text{unpredictable}\\\text{noise}}}.$$

Restoration of the unknown wavefront from given sensor measurements and further calculation of optimal mirror deformation is an **inverse problem**.

Fourier filtering optical system

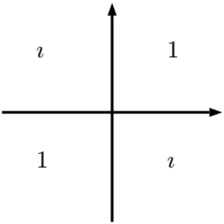


Credit: O. Fauvarque

$$I(x,y) = \left|\mathcal{F}^{-1}\left(OTF \cdot \mathcal{F}\left(\mathcal{X}_{\Omega} e^{-i\Phi}\right)\right)\right|^2$$

$\imath$Quad wavefront sensor:
a new Fourier-based WFS

- derived from the 4-quadrants coronagraph
- focal plane is devided into 4 quadrants around the origin
- each quadrant is $\pi/2$ shifted with its 2 neighbours

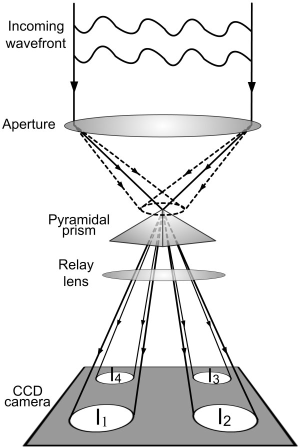


Transparency function of the 4-quadrants sensing mask

Pyramid wavefront sensor:
baseline for many future ELT instruments

Extremely Large Telescope, Very Large Telescope, and the Pyramids of Giza, Credit: ESO

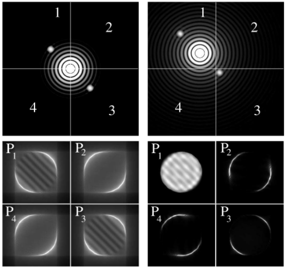
Pyramid wavefront sensor



Credit: Iu. Shatokhina

- (oscillating) pyramidal prism
- splits light into 4 distinct directions
- 4 intensities are measured
- those can be combined to 2 signals

Pyramid wavefront sensor (PWFS) measurements

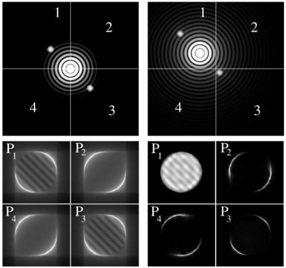


Credit: O. Guyon

<u>modulation of the beam:</u>

- increased linear range
- reduced sensitivity

Pyramid wavefront sensor (PWFS) measurements



Credit: O. Guyon

modulation of the beam:

- increased linear range
- reduced sensitivity

$$s_x(x,y) = \frac{[I_1(x,y) + I_2(x,y)] - [I_3(x,y) + I_4(x,y)]}{I_0}$$

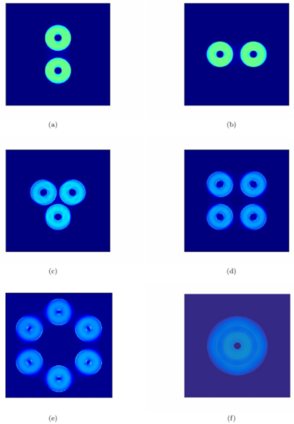
$$s_y(x,y) = \frac{[I_1(x,y) + I_4(x,y)] - [I_2(x,y) + I_3(x,y)]}{I_0}$$

I_0 – average intensity per subaperture

Different concepts of a pyramid sensor

different configurations of prism:

a) 2-sided (roof) prism
b) 2-sided (roof) prism
c) 3-sided
d) 4-sided
e) 6-sided
f) cone



Credit: B. Engler

Problem description

Pyramid sensor measuring process:

$$P\Phi = s$$

(Φ ... incoming wavefront, s ... pyramid sensor measurements)

interaction-matrix-based:

$\overline{\boldsymbol{P}}$ ... calibrated matrix

This is the benchmark with respect to quality and speed.

model-based:

$\boldsymbol{P}$ ... non-linear operator

This is the basis of the new methods.

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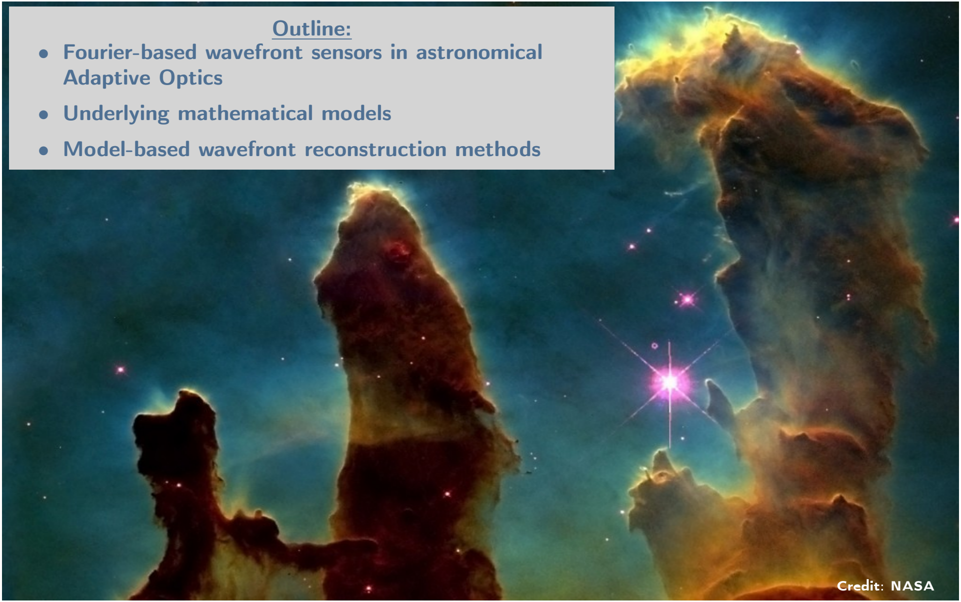
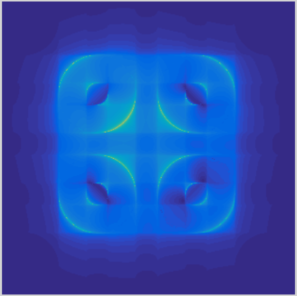


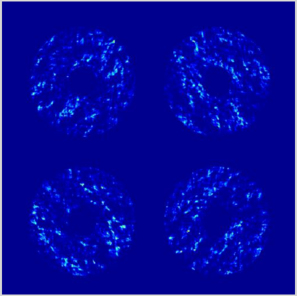
Credit: NASA

How do we model it mathematically?

phase mask model
underlying model
in forward simulations



transmission mask model
underlying model
of reconstructors



Credit: Iu. Shatokhina

PWFS transmission mask models

Theorem

The relation between pyramid wavefront sensor data **with circular modulation** and the incoming phase following the transmission mask model is given by

$$s_x^c(x,y) = \frac{1}{2\pi}\mathcal{X}_\Omega(x,y)\int_{\Omega_y}\frac{\sin[\Phi(x',y)-\Phi(x,y)]\mathbf{J_0}[\alpha_\lambda(\mathbf{x'}-\mathbf{x})]}{x-x'}\,dx'$$

$$+\frac{1}{2\pi^3}\mathcal{X}_{\Omega_y}(x,y)\ \ p.v.\int_{\Omega_y}\int_{\Omega_x}\int_{\Omega_x}\frac{\sin[\Phi(x',y')-\Phi(x,y'')]\mathbf{f}(x'-x,y'-y'')}{(x-x')(y-y')(y-y'')}\,dy''dy'dx',$$

$$\mathbf{f}(\tilde{\mathbf{x}},\tilde{\mathbf{y}}) := \frac{\mathbf{1}}{\mathbf{T}}\int_{-\mathbf{T}/2}^{\mathbf{T}/2}\cos[\alpha_\lambda\tilde{\mathbf{x}}\sin(\mathbf{2}\pi\mathbf{t}/\mathbf{T})]\cos[\alpha_\lambda\tilde{\mathbf{y}}\cos(\mathbf{2}\pi\mathbf{t}/\mathbf{T})]\,\mathbf{dt}$$

and s_y accordingly.

J_0 ... zero order Bessel function of first kind

α_λ ... modulation parameter

Pyramid WFS without modulation

Theorem

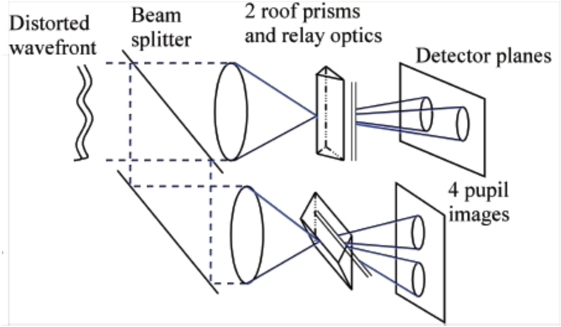
The measurements of the PWFS without modulation are given by

$$s_x(x,y) = \frac{1}{2\pi}\mathcal{X}_\Omega(x,y)\int\limits_{\Omega_y} \frac{\sin[\Phi(x',y)-\Phi(x,y)]}{x-x'}\,dx'$$
$$+\frac{1}{2\pi^3}\mathcal{X}_{\Omega_y}(x,y)\ p.v.\int\limits_{\Omega_y}\int\limits_{\Omega_x}\int\limits_{\Omega_x}\frac{\sin[\Phi(x',y')-\Phi(x,y'')]}{(x-x')(y-y')(y-y'')}\,dy''dy'dx',$$

assumptions:

- **roof wavefront sensor**

Roof WFS approximation



Credit: C. Vérinaud

Pyramid WFS without modulation

Theorem

The measurements of the PWFS without modulation are approximated by

$$s_x(x,y) \quad \sim \quad \frac{1}{2\pi}\mathcal{X}_\Omega\,(x,y)\int\limits_{\Omega_{\boldsymbol{y}}} \frac{\sin[\Phi(x',y)-\Phi(x,y)]}{x-x'}\,dx'$$

assumptions:

- **roof wavefront sensor**
  - substitute four-sided prism by two orthogonally placed two-sided prisms
  - two signal sets s_x and s_y are independent and contain information about Φ only in x- and only in y-direction correspondingly
- small wavefront distortions (as expected in closed loop), $\Phi \ll 1 \to \sin\Phi \simeq \Phi$
- without second term

Pyramid WFS without modulation

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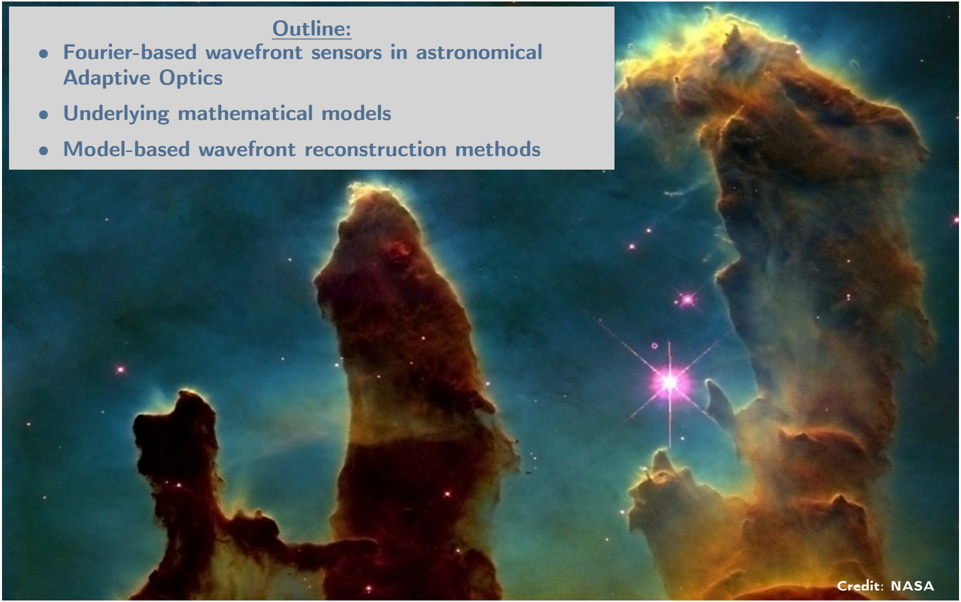


Credit: NASA

Preprocessed CuReD (P-CuReD)

assumption: closed loop AO.

P-CuReD = data preprocessing + CuReD

two-step method:

- **data preprocessing:** transform the PWFS data to SH-like data according to the analytical relation in the Fourier domain.
- **CuReD:** apply the CuReD to the modified data.
  (CuReD is a very efficient reconstructor for SH WFS, linear complexity)

Step 1: data preprocessing [Iu. Shatokhina]

representation of the measurements in the Fourier domain

$$(\mathcal{F}s_{pyr})(u) = (\mathcal{F}\Phi)(u) \cdot g_{pyr}(u) \cdot \operatorname{sinc}(du)$$

$$(\mathcal{F}s_{sh})(u) = (\mathcal{F}\Phi)(u) \cdot g_{sh}(u) \cdot \operatorname{sinc}(du)$$

u – spatial frequency, d – subaperture size.

Fourier domain relation between the two sensors

$$(\mathcal{F}s_{sh})(u) = (\mathcal{F}s_{pyr})(u) \cdot g_{sh/pyr}(u), \qquad g_{sh/pyr}(u) := \frac{g_{sh}(u)}{g_{pyr}(u)}.$$

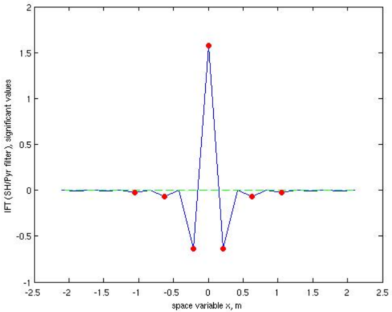
Fourier convolution theorem → relation between the two sensors in the space domain

$$s_{sh}(x,y) = \frac{1}{\sqrt{2\pi}} s_{pyr}(\cdot, y) * \underbrace{\left(\mathcal{F}^{-1} g_{sh/pyr}\right)(\cdot)}_{p_{sh/pyr}(x)}.$$

Step 1: data preprocessing [Iu. Shatokhina]

convolve data set with 1d kernel $p_{sh/pyr}$

$$s_{sh}(x,y) = \frac{1}{\sqrt{2\pi}} s_{pyr}(\cdot, y) * p_{sh/pyr}(\cdot).$$



computationally very cheap, highly parallelizable and pipelinable

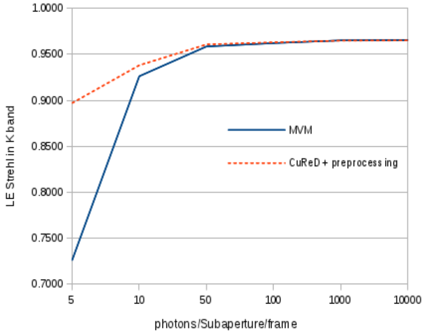
Step 2: application of CuReD [M. Rosensteiner]

28./29. September 2012: CuReD on sky, Herschel telescope

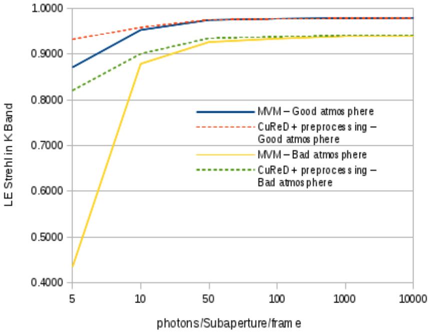
- Las Palmas, Canary Islands (Spain), Roque de los Muchachos (2344m)
- 4.2 m mirror diameter
- successful CuReD-tests of the University of Durham, code from AAO team

Quality and speed performance

LE Strehl in K band: MVM and P-CuReD vs. the detected NGS photon flux.



ESO median atmosphere



ESO bad/good atmospheres

P-CuReD on the LOOPS bench

- application of a model-based reconstructor on LOOPS
- closed the loop for both PWFS with & without modulation
- reconstruction quality comparable to approach with calibrated MVM

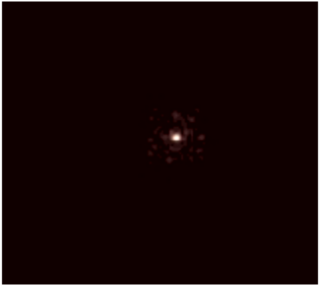


Figure: closing the loop on LOOPS with P-CuReD

Recent ELT adaptions: telescope spiders

- pupil fragmentation & **disconnectedness of data** (wavefront information)
- differential piston effects between the segments
- if not properly handled extremely poor wavefront reconstruction

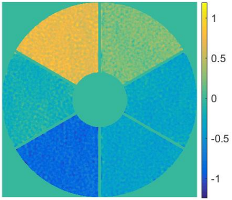


Figure: residual phase in radians (K-band)

How much quality do we loose in the presence of spiders?
How can we make existing reconstruction methods feasible?

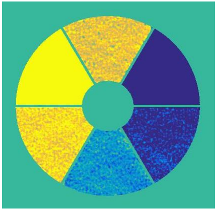
Split Approach

Split Approach =

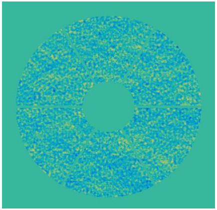
piston-free WF reconstruction + direct segment piston reconstruction

Requests:

- compoundable with all existing reconstruction methods
- providing high reconstruction quality
- low computational complexity



Split Approach $\longrightarrow$



$\imath$Quad wavefront sensor:
first numerical results - "optical" linear Landweber iteration

minimize least-squares functional

$$J(\Phi) := ||mI(\Phi_?) - Q(\Phi_?)||^2_{\mathcal{L}_2} \qquad \rightarrow \qquad \text{min}$$
$$J'(\Phi) = Q^*(Q\Phi_? - mI(\Phi_?))$$

linear iterative Landweber algorithm:

$$\Phi_{k+1} = \Phi_k + \alpha Q^* \left(mI(\Phi_?) - Q(\Phi_k) \right)$$

$\Phi_?$ is the phase-to-be-measured/reconstructed, Q the wavefront sensor operator and $mI(\Phi_?)$ the corresponding meta-intensity.

$\imath$Quad wavefront sensor:
first numerical results - "optical" linear Landweber iteration

small phases $\rightarrow$ linear intensity:

$$Q(\phi) \approx \frac{1}{\epsilon} mI(\epsilon\Phi) \qquad \text{with} \qquad \epsilon << 1$$

adjoint:

$$Q = Q^*$$

linear iterative Landweber algorithm:

$$\Phi_{k+1} = \Phi_k + \alpha Q^* \left(mI(\Phi_?) - Q(\Phi_k) \right)$$

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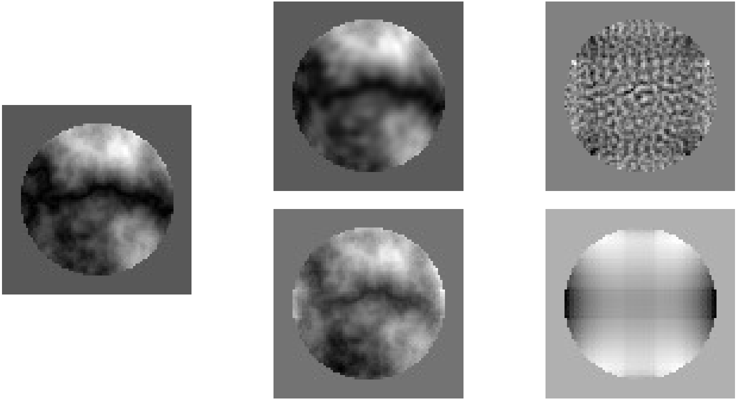
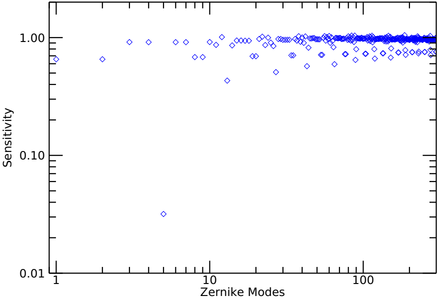
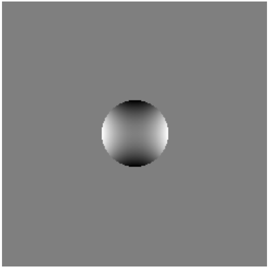


Left to right: incoming, reconstructed, residual phase.
Top: interaction-matrix-based inversion. Bottom: linear Landweber iteration.

ıQuad wavefront sensor:
the unseen mode of the ıQuad





Sensitivity curve for the ıQuad sensor wrt. Zernike modes (left) and the mode (vertical astigmatism) with the low sensitivity (right).

Conclusions

- model-based reconstructors are promising alternatives to MVM approaches
  - no calibration of interaction matrix needed
  - reconstruction qualities are highly comparable (in end2end simulations & on testbed)
- adaptions of existing methods to ELT effects is necessary
- possibilities to combine mathematical theory and optical considerations

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- possibilities to combine mathematical theory and optical considerations

Outlook:

- derivation of a general framework for reconstructors of Fourier-based wavefront sensors
- comparison of "optical" reconstruction approaches and model-based implementations

Thank you for your attention

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