Efficient model-based wavefront control for coronagraphy using nonlinear optimization

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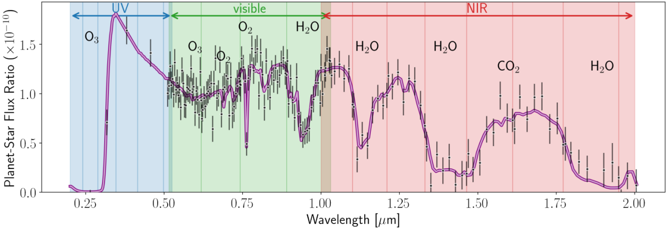
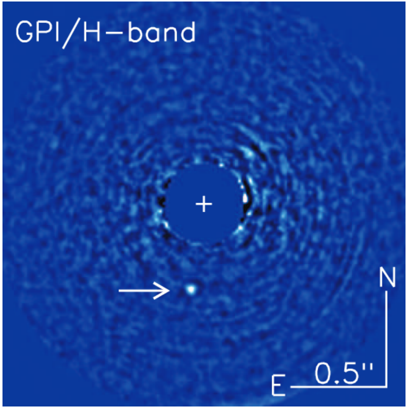
The search for life on extrasolar planets

- More than 4000 exoplanets have been discovered since 1992 (∼2300 by Kepler)
  - Mostly via transit timing, radial velocities (indirect)
  - Limited spectral information via transit spectroscopy
- Direct imaging enables full spectroscopic analysis in search for indicators of life (c.f. H_2O, CO_2)



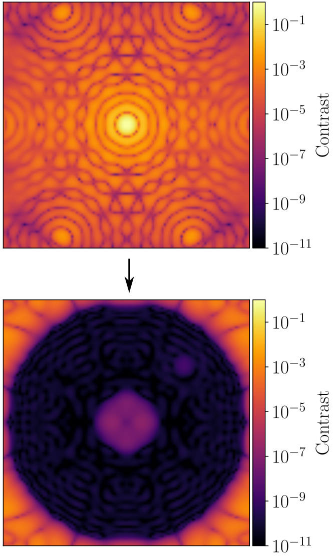
Source: https://asd.gsfc.nasa.gov/luvoir/



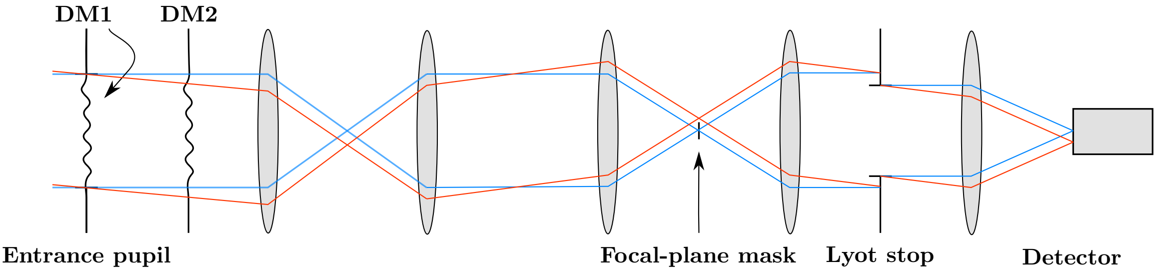
Source: Macintosh et al., Science **350**(6256), 64–67 (2015)

Direct imaging of exoplanets: coronagraphy

- **Goal**: suppress light from bright on-axis star, transmit signal from faint off-axis planet inside **dark zone**
- Key metrics:
  - **Flux ratio**: $\frac{\text{Total flux from planet entering telescope}}{\text{Total flux from star entering telescope}}$
    - Hot Jupiter-like planet in NIR: 10^{-6}
    - Earth-like planet in visible: $\lesssim 10^{-10}$
  - **Contrast**: $\frac{\text{Intensity at } (\theta_x, \theta_y) \text{ from star with coronagraph}}{\text{Peak intensity from star without coronagraph}}$
- Angular separation for rocky planets around nearby stars ~ 0.1 arcsec
- Requires high-precision optical instrumentation + active wavefront sensing/control using deformable mirrors (DMs)

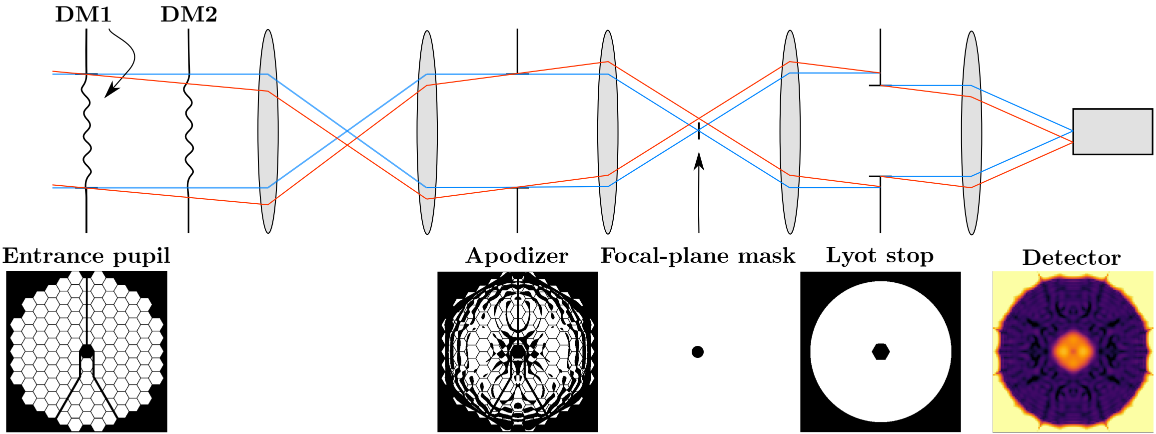


Lyot coronagraphy



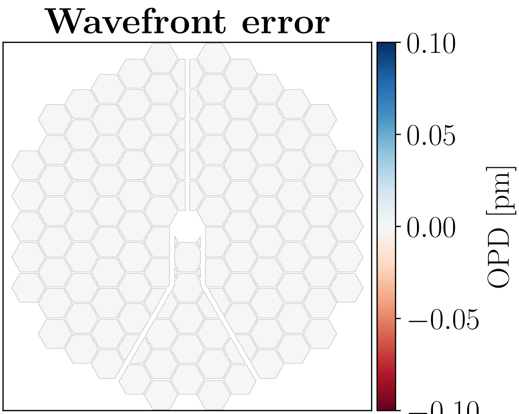
- **Focal-plane mask (FPM)** blocks core of on-axis stellar PSF, transmits off-axis planetary PSF
- **Lyot stop** suppresses sidelobes of on-axis stellar PSF
- **Deformable mirrors** (DM1/DM2) compensate for amplitude and phase aberrations

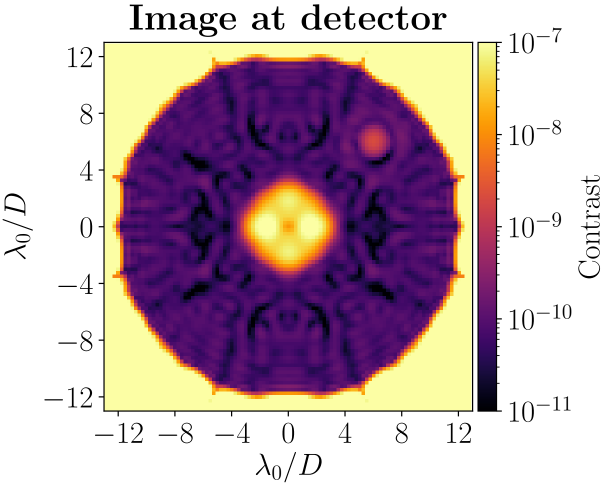
Apodized pupil Lyot coronagraph (APLC)



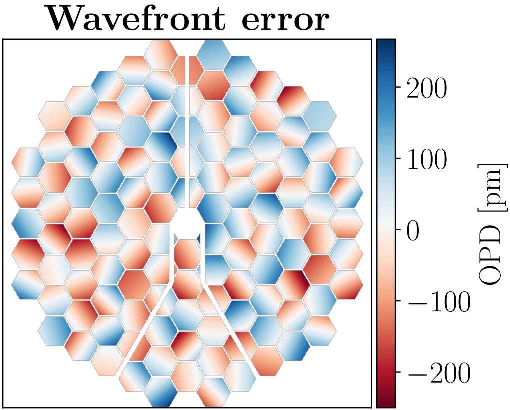
- **Apodizer** suppresses unwanted diffraction from pupil features (segment gaps, struts)

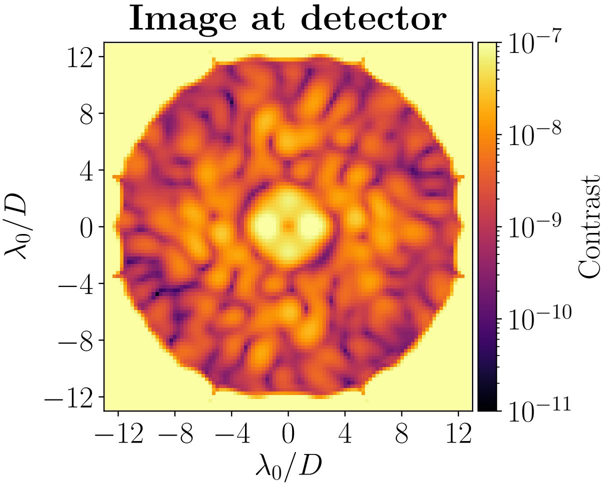
Planet detection: no wavefront error





Planet detection: 50 pm RMS per-segment piston/tip/tilt





Wavefront sensing and control

Aberrated E-field → $\mathbf{E}_{\mathrm{ab},k}$

E-field from DMs → $\mathbf{E}_{\mathrm{DM},k}(\mathbf{a}_k)$

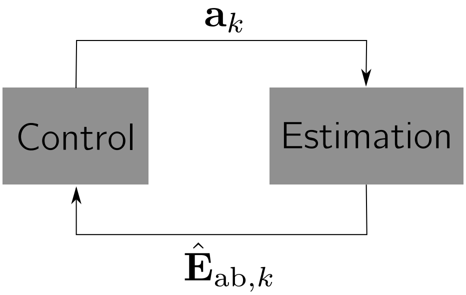
$$\mathbf{E}_{D,k} \approx \mathbf{E}_{\mathrm{ab},k} + \mathbf{E}_{\mathrm{DM},k}(\mathbf{a}_k)$$

Total E-field at detector → $\mathbf{E}_{D,k}$

Iteration index → k

DM actuator commands → $\mathbf{a}_k$

- **Closed-loop wavefront sensing and control** iteratively minimizes starlight in dark zone
- **Two deformable mirrors** (one in-pupil and one out-of-pupil) correct amplitude and phase aberrations over symmetric dark zone



Model-based wavefront control

- **Goal**: choose DM commands $\mathbf{a}_k$ to drive $\mathbf{E}_{D,k}$ to zero as $k \to \infty$
- Use numerical model to predict $\mathbf{E}_{\mathrm{DM},k}(\mathbf{a}_k)$, solve inverse problem for $\mathbf{a}_k$
- Two common inverse problems:

1. **Stroke minimization (SM)**[1]:

$$\underset{\mathbf{a}_k}{\arg\min} \quad \mathbf{a}_k^T \mathbf{a}_k \quad \text{subject to} \quad \overbrace{\mathbf{E}_{D,k}^\dagger \mathbf{E}_{D,k}}^{\text{Integrated dark-zone intensity}} \leq \underbrace{I_{T,k}}_{\text{Target}} \tag{1}$$

2. **Electric field conjugation (EFC)**[2,3]:

$$\underset{\mathbf{a}_k}{\arg\min} \quad \left(\mathbf{E}_{D,k} - \mathbf{E}_{T,k}\right)^\dagger \left(\mathbf{E}_{D,k} - \mathbf{E}_{T,k}\right) + \mathbf{a}_k \mathbf{\Gamma}^T \mathbf{\Gamma} \mathbf{a}_k \tag{2}$$

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[1] L. Pueyo et al., *Appl. Opt.* **48**, 6296–6312 (2009)
[2] A. Give'on et al., *Proc. SPIE* **7440**, 74400D (2009)
[3] SM and EFC are equivalent when $\mathbf{E}_{T,k} = 0$ and $\mathbf{\Gamma} = \alpha \mathbf{I}$

Stroke minimization

- Method of Lagrange multipliers: find critical point of Lagrangian function

$$\mathcal{L}_k \triangleq \mathbf{a}_k^T \mathbf{a}_k + \mu(\mathbf{E}_{D,k}^{\dagger} \mathbf{E}_{D,k} - I_{T,k}) \tag{3}$$

- **Conventional approach**:

1. Use numerical model to evaluate $\mathbf{G}_k \triangleq \dfrac{\partial \mathbf{E}_{D,k}}{\partial \mathbf{a}_k}$. Then

$$\mathbf{a}_k^*(\mu) = -\left[\frac{1}{\mu} + \Re\left\{\mathbf{G}_k^{\dagger}\mathbf{G}_k\right\}\right]^{-1} \Re\left\{\mathbf{G}_k^{\dagger}\widehat{\mathbf{E}}_{\mathrm{ab},k}\right\} \tag{4}$$

2. Generate family of solutions with different μ
3. Choose smallest μ such that $\mathbf{E}_{D,k}^{\dagger}\mathbf{E}_{D,k} \leq I_{T,k}$[4]

[4]**Reminder**: $\mathbf{E}_{D,k} \approx \mathbf{E}_{\mathrm{ab},k} + \mathbf{E}_{\mathrm{DM},k}(\mathbf{a}_k)$

Stroke minimization

$$\mathbf{a}_k^*(\mu) = -\left[\frac{1}{\mu} + \Re\left\{\mathbf{G}_k^\dagger \mathbf{G}_k\right\}\right]^{-1} \Re\left\{\mathbf{G}_k^\dagger \widehat{\mathbf{E}}_{\mathrm{ab},k}\right\} \tag{5}$$

- **Problem**: have to calculate matrix-valued derivative $\mathbf{G}_k \in \mathbb{C}^{N_{\mathrm{pix}} \times N_{\mathrm{act}}}$
  - **Computationally expensive**
  - Usually just compute $\mathbf{G}_0$ before start of experiment and reuse for all iterations $\rightarrow$ reduces speed of convergence to high contrast
  - For multi-wavelength control, need to compute separate $\mathbf{G}$ matrix for each controlled wavelength
- Can we do better?

The solution: reverse-mode algorithmic differentiation (RMAD)[6]

- Efficient, analytical differentiation of **numerical algorithms**
- **Main idea**: given **forward model** consisting of sequence of N differentiable operations $x_n = f_n(x_{n-1})$ with x_N scalar, construct **adjoint model** that evaluates $\partial x_N / \partial x_n \triangleq \overline{x}_n$
- **Gradient propagation rule**:

$$\text{if} \quad x_n = f_n(x_{n-1}) \tag{6}$$

$$\text{then} \quad \overline{x}_{n-1} = \underbrace{\left(\left. \frac{\partial f_n}{\partial x_{n-1}} \right|_{x_{n-1}} \right)^{\dagger}}_{\text{Adjoint of Jacobian matrix of } f_n \text{, evaluated at } x_{n-1}} \overline{x}_n \tag{7}$$

- Cost of evaluating gradient $\sim$ cost of evaluating forward model (**cheap gradient principle**[5])

[5] A. S. Jurling and J. R. Fienup, *J. Opt. Soc. Am. A* **31**(7), 1348–59 (2014)
[6] Also known as **backpropagation algorithm** in machine learning with neural networks

Reverse-mode algorithmic differentiation

- **Example (phase retrieval)**: retrieve Zernike coefficients $\mathbf{a}$ that best explain data $\mathbf{D}$

Forward model	Adjoint model
$\phi = \mathbf{Za}$	$\bar{\mathbf{I}} = 2(\mathbf{I} - \mathbf{D})$
$\psi = \exp\left\{ i \frac{2\pi}{\lambda} \phi \right\}$	$\overline{\mathbf{M}} = 2\mathrm{M} \circ \bar{\mathbf{I}}$
$\mathbf{M} = \mathrm{FFT}\{\psi\}$	$\overline{\psi} = \mathrm{IFFT}\left\{ \overline{\mathbf{M}} \right\}$
$\mathbf{I} = \lvert \mathbf{M} \rvert^2$	$\overline{\phi} = \frac{2\pi}{\lambda} \Im\left\{ \overline{\psi} \circ \psi^* \right\}$
$E = \lVert \mathbf{I} - \mathbf{D} \rVert^2$	$\bar{\mathbf{a}} = \mathbf{Z}^T \overline{\phi}$

1. Given value of $\mathbf{a}$, evaluate variables in forward model (ϕ, ψ, $\mathbf{M}$, $\mathbf{I}$, E)
2. Insert into adjoint model and evaluate adjoint variables ($\bar{\mathbf{I}}$, $\overline{\mathbf{M}}$, $\overline{\psi}$, $\overline{\phi}$, $\bar{\mathbf{a}}$)

Proposed algorithm[7]

- Stroke minimization, but with quadratic contrast penalty instead of linear:

$$J_k = \mathbf{a}_k^T \mathbf{a}_k + \mu \left(\mathbf{E}_{D,k}^\dagger \mathbf{E}_{D,k} - I_{T,k} \right)^2 \tag{8}$$

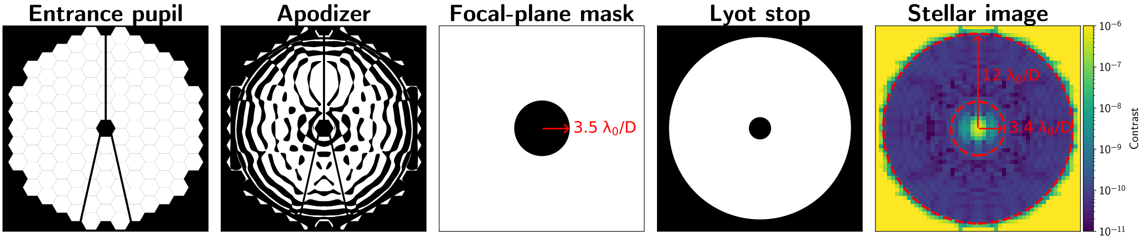
  - Quadratic penalty: enforces contrast target without line search on μ

- Use gradient-based nonlinear optimization with RMAD gradient $\overline{\mathbf{a}}_k$ to minimize J_k in each control iteration
- **Advantages**:
  - Don't need to calculate $\mathbf{G}_k$ at all $\rightarrow$ { Massive reduction in up-front computation; Easier to update model between iterations }
  - Only **vector-valued derivatives** handled during gradient computation $\rightarrow$ better scaling with actuator count, dark zone size

[7] S. D. Will, T. D. Groff, and J. R. Fienup, *JATIS* (2020, under review)

Simulations: overview

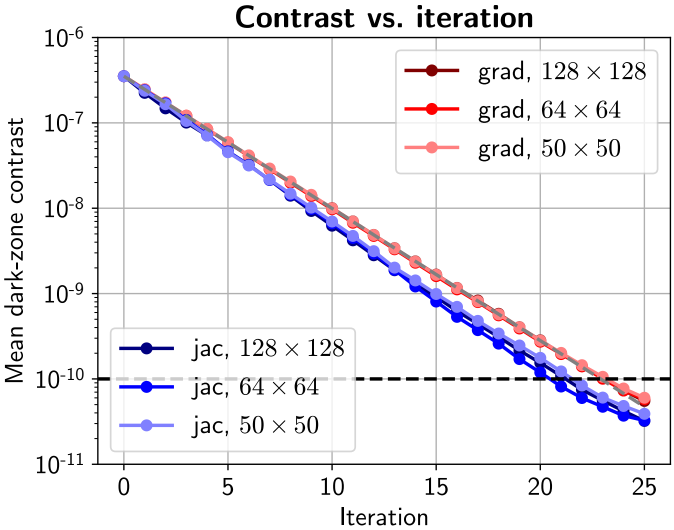
- Compared proposed gradient-based method to Jacobian-based algorithm
- Small-angle APLC design[8] submitted to 2020 Astrophysics Decadal Survey for proposed LUVOIR mission[9]
- Three different MEMS DM formats: 50×50, 64×64, 128×128



[8] Courtesy of R. Soummer

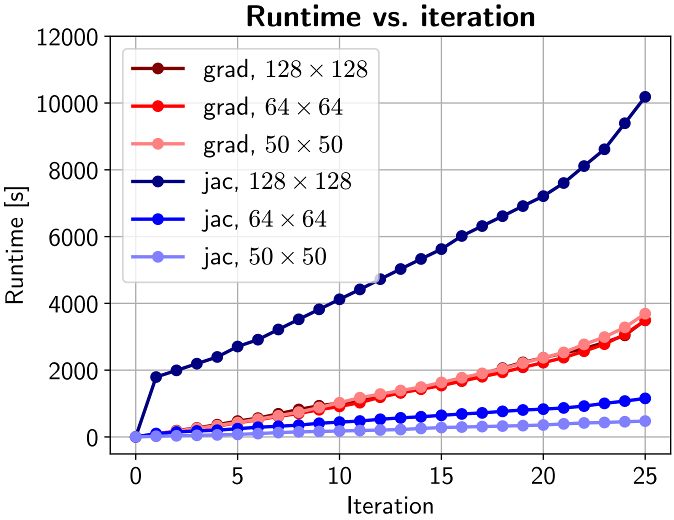
[9] https://asd.gsfc.nasa.gov/luvoir/

Simulation results



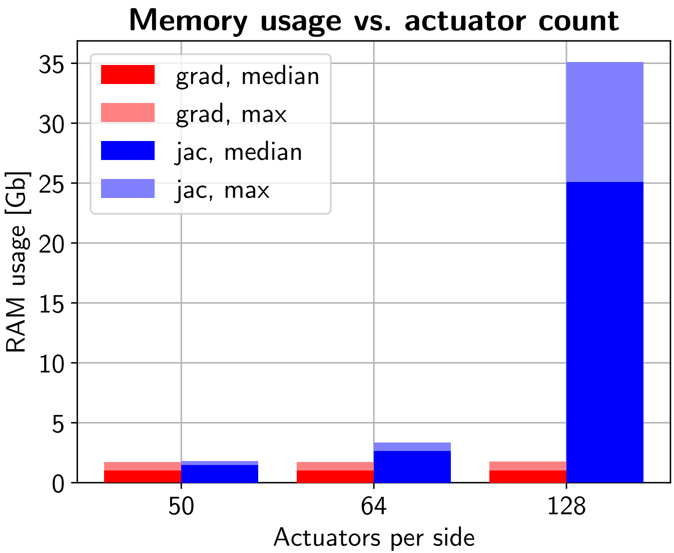
- Both algorithms converge as expected after 25 iterations
- Jacobian-based algorithm overshoots slightly in optimistic direction due to Lagrange multiplier line search

Simulation results



- Jacobian-based algorithm faster for lowest actuator counts, but very slow for 128×128 case
- Proposed algorithm runtime invariant to actuator count
- Results shown do **not** factor in time cost of precomputing $\mathbf{G}_0$

Simulation results



- Memory consumption of proposed algorithm invariant to actuator count
- At 128×128 actuators per DM (baseline for LUVOIR mission), Jacobian-based algorithm consumes $10\times$ more memory

Future work

- Laboratory demonstrations
- **Dynamically updated model**: use data from low-order wavefront sensor (LOWFS) in each control iteration to capture time-varying aberrations
- **Adaptive control**:
  - Compute gradients with respect to model parameters (DM influence function, pupil shear, pupil transmittance, etc.) and use control history to tune model
  - Alternate between contrast improvement and model improvement