Phasing of a Large Segmented Mirror with a Pyramid
Simulation vs. Experiment

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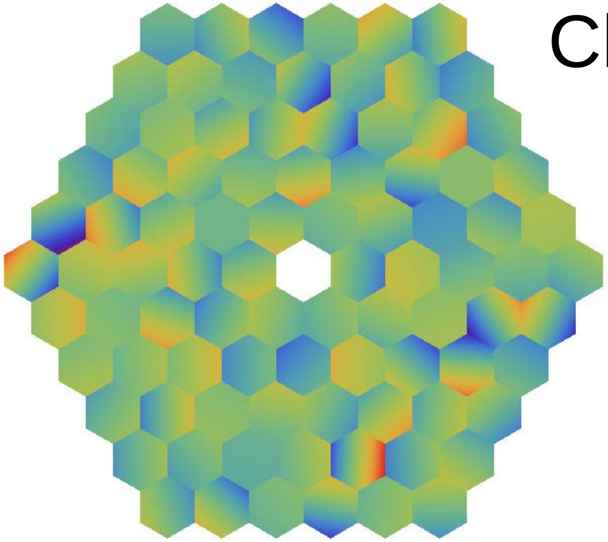
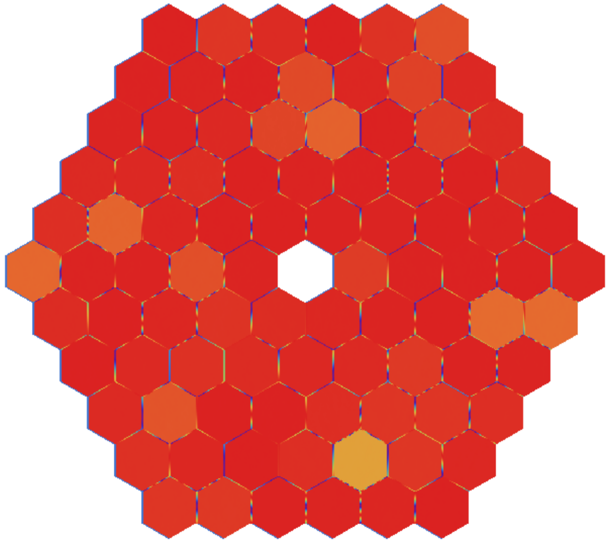




GRD Seminar, LAM

22 Oct 2020

Test Case: Study of Segment Phasing Scenarios Objective and Assumptions

- Phase large segmented mirrors for diffraction-limited observation (hundreds of segments, full pupil at the same time)
- Assume that segments are already coarsely stacked and piston errors are down to a few tens of waves
- Goal: Phase the mirror with a single WFS in as few exposures and position corrections as possible (one?), without need for (strong) AO
- Must be robust and registration/measurement error tolerant
- Neglecting detector noise, sky background, segment-to-pupil distortion for the moment; assuming bright star in long exposure (30–50 s)
- **This is a technical study**: Evaluating trades between different wavefront sensors and options for phase reconstruction methods

Wavefront Sensor Wishlist

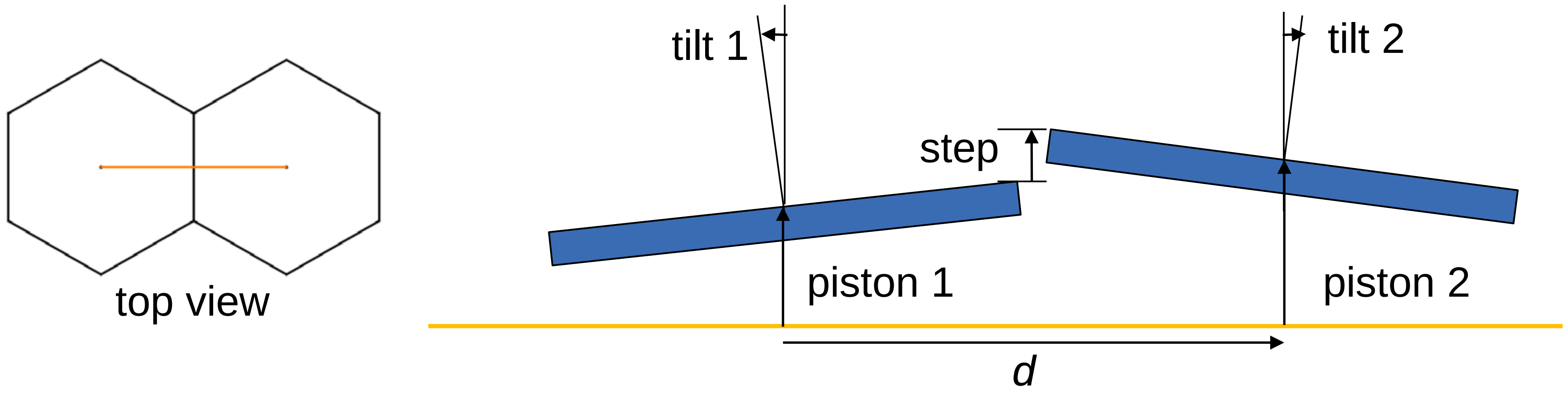
- Sense steps *and* segment tip/tilt in parallel; robust to petaling
- Simple, linear response function (weak saturation and low cross-talk)
- Keep structures resolved (no smearing) even in good seeing
- Can be imaged with a reasonable number of detector pixels
- Easy registration: Does not require accurate optical pupil alignment

Some well-known candidates:

  - Shack-Hartmann (with custom lenslet geometry?, in APE: *SHAPS*)
  - Phase contrast WFS (phase contrast mask in the focal plane, *ZEUS*)
  - Ext. Hartmann phase mask WFS (Ronchi grating/shearing like *Phasics SID-4*)
  - Pyramid WFS with modulation (order 500–1000 pixels across pupil, *PYPS*)

Geometric Equations

- ELT M1 segment state variables: {piston,tip,tilt} × 798 = 2394
- Each edge between two segments yields one equation:



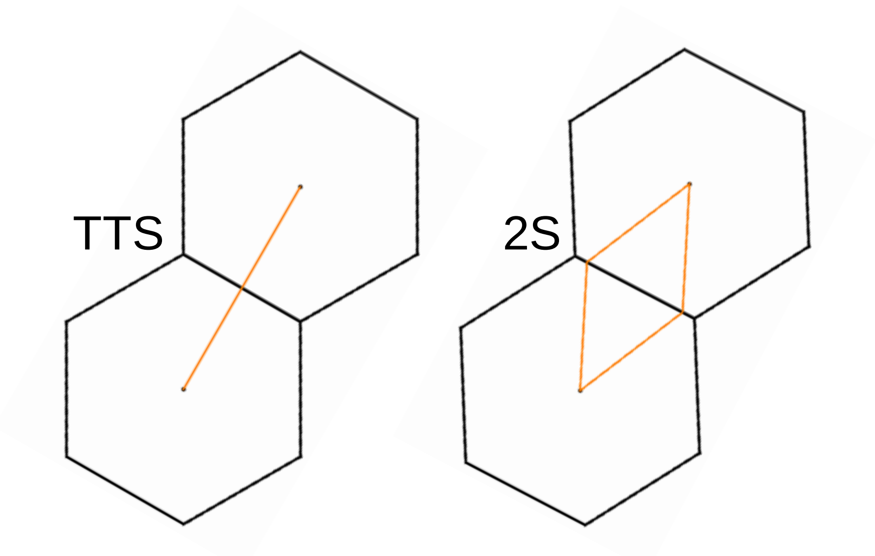
- Link two pistons and tilts via the segment step:

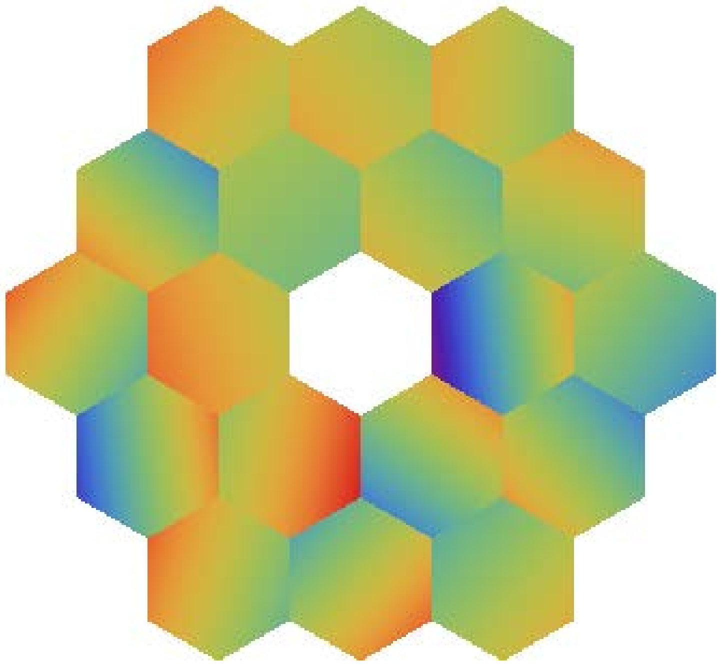
$$\text{step}_{1\rightarrow 2} = \text{piston}_2 - \text{piston}_1 - (\text{tilt}_1 + \text{tilt}_2)\, d/2$$

➔ Entire mirror M1: Highly overdetermined linear equation system

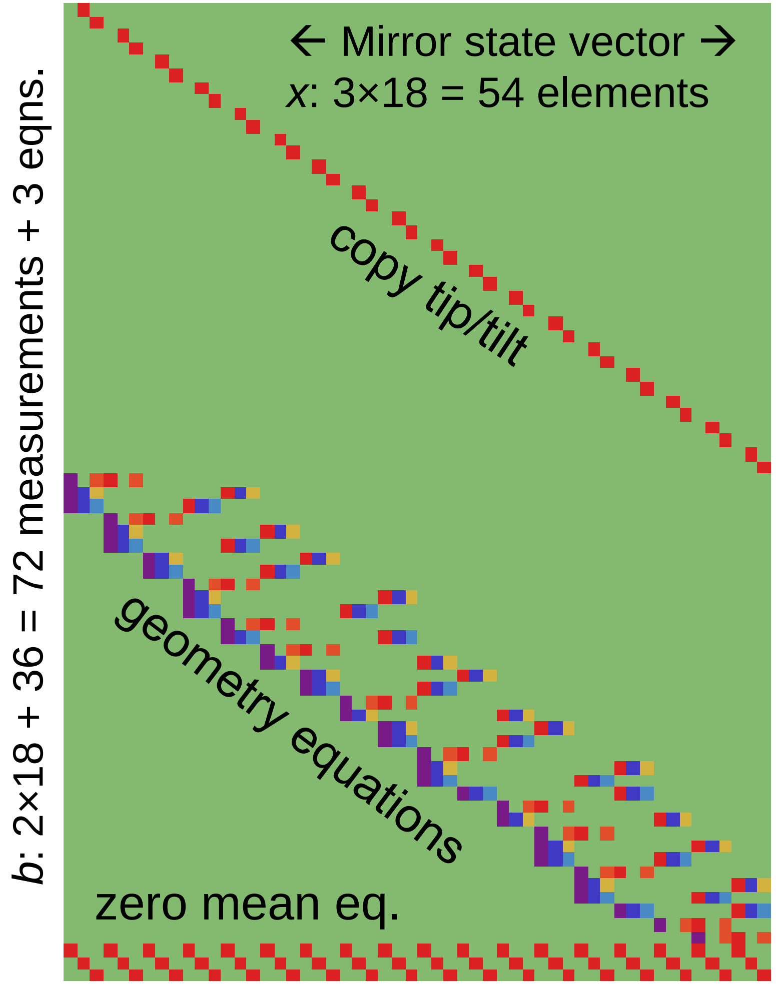
Sensing Types

- Can distinguish several sensing types, e.g.
  - Measure tip/tilt + single step per edge (TTS)
  - Measure 2 steps per edge (2S)





TTS design matrix ***A*** for 18 segments (JWST)



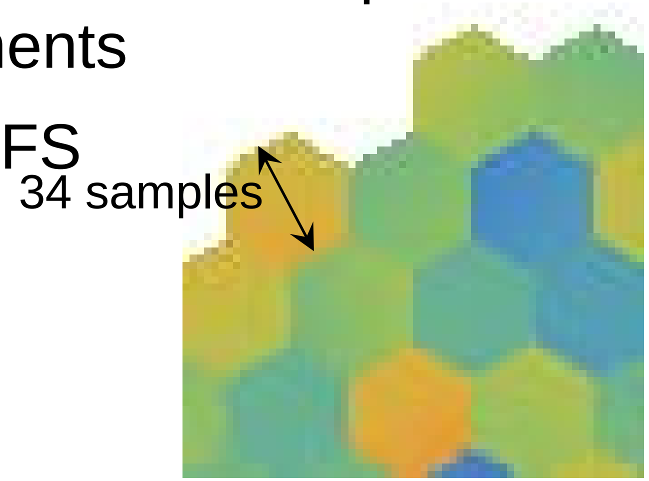
- Minimize $r = \boldsymbol{A} \cdot x - b$,

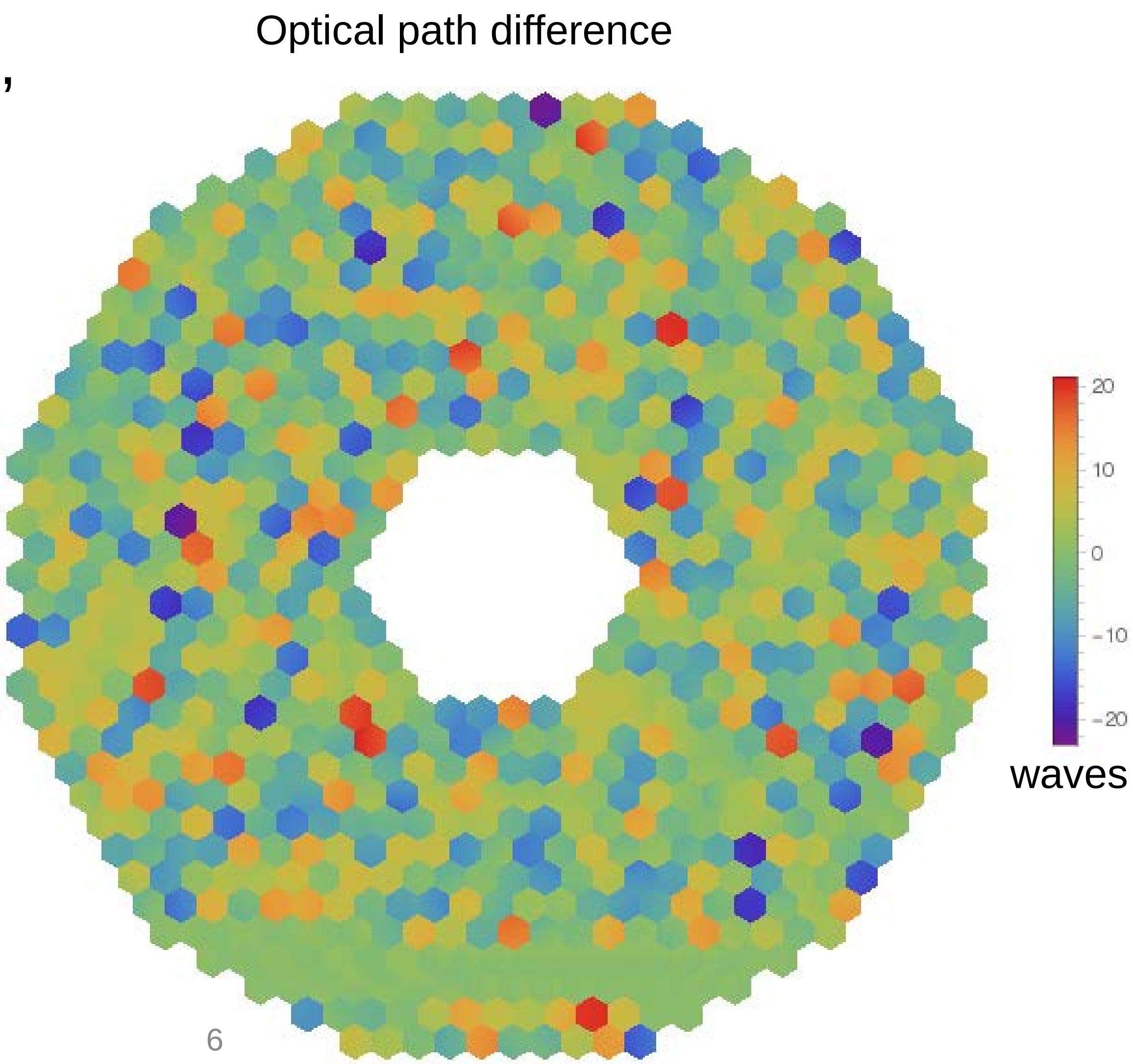
Redundancy degrees JWST: 18, TMT: 894, ELT: 1467

Simulations

- Monochromatic in NIR, physical optics with FFT size 1176^2, non-elongated point source, average over 4000 independent phase screens
- 798 ELT-size hexagonal segments (1.22 m edge-to-edge), 2 edges aligned with pixel grid
- Gaussian random distribution of tip/tilt and piston misalignments
- Study the pyramid WFS
- Resolution:

3.6 cm, angle: 8.6 mas





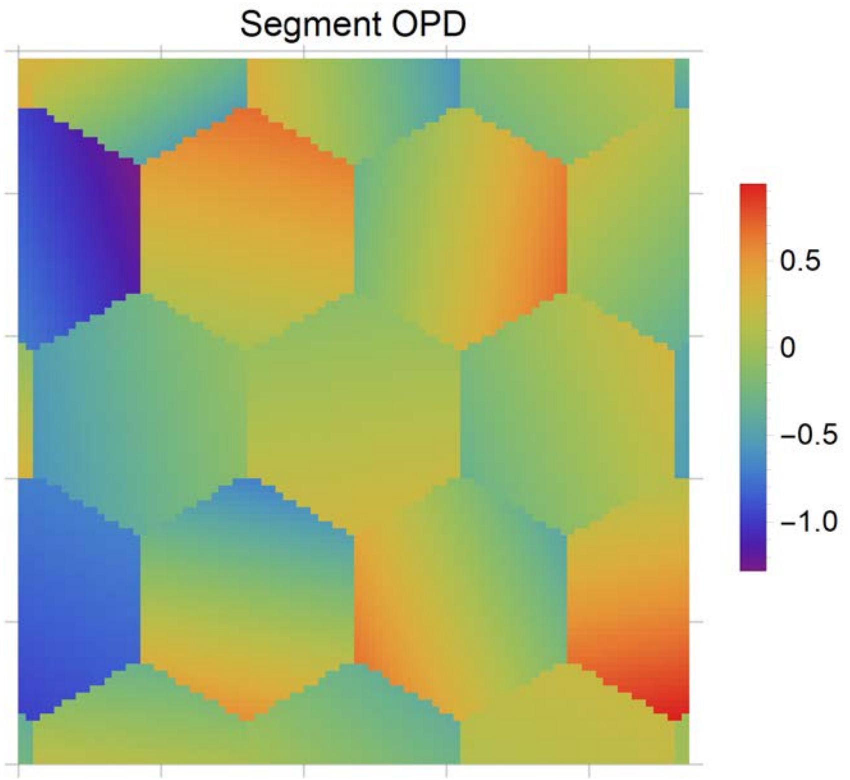
Pyramid Phase Step Response

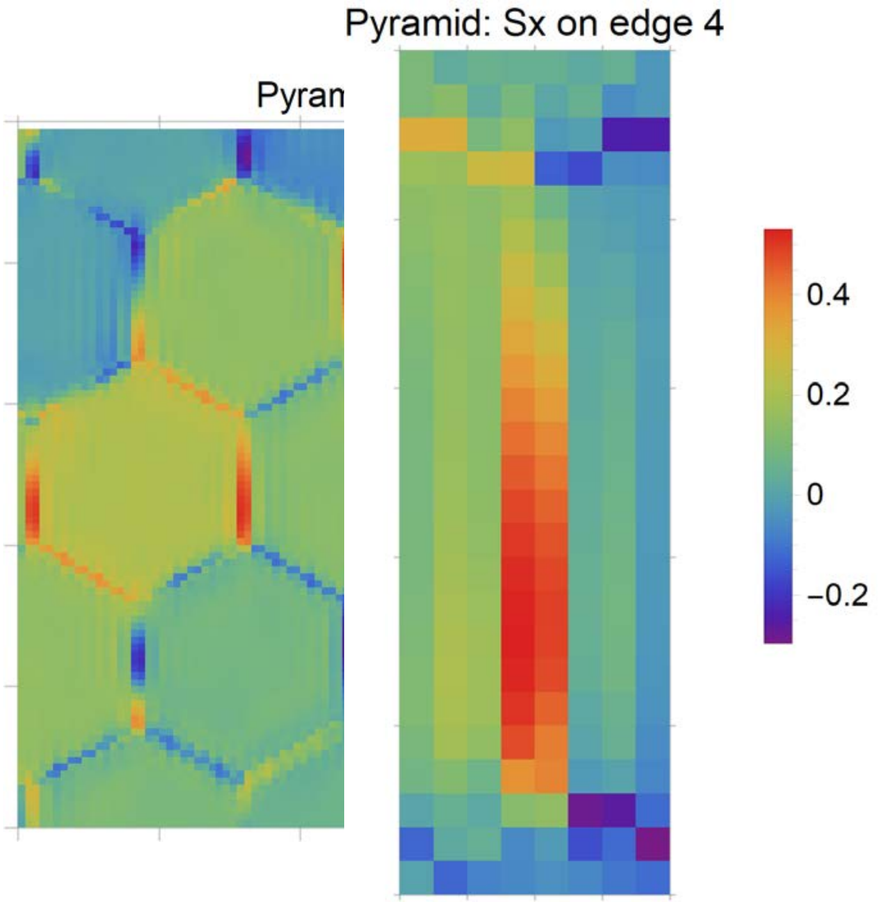
- The pyramid WFS (PWFS) can sense phase discontinuities ("steps" $\Delta\varphi$) in the pupil plane, e.g. caused by segment misalignment or petaling
- The PWFS phase step response, expressed as the slope S_x or S_y across the step, is a tent-like single peaked function (*i.e.*, well localized)
- To first order, the peak height near the step equals $S_{peak} = \sin(\Delta\varphi)/2$
- S_{peak} and the width of the "tent" $S(x)$ decrease with pyramid modulation radius and turbulence strength
- S_{peak} also decreases with the width w of the gap between the phase patches (if any). The gap is a dark region, e.g. between M1 segments (w = 6 mm in the ELT) or within the spider shadows (ELT: w = 530 mm)
- The latter effect is detrimental to petaling sensing
- S_{peak} will also diminish when working with extended sources, e.g., (elongated) laser guide stars

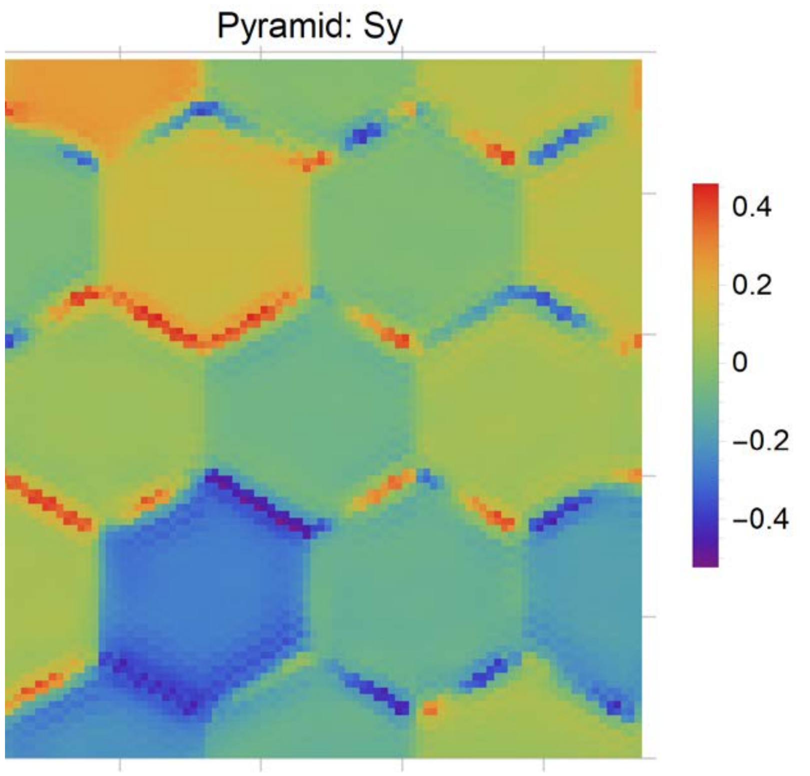
Modulated Pyramid WFS

- 1780 nm (H-band), averaged over 4000 phase screens, X- and Y-slopes (S_x, S_y)
- Seeing: 0.67” at 500 nm, IQ: 0.37”, r_0 : 602 mm (14.6 samples)

PWFS modulation radius:
0.47 " = 1.6 × λ/edge



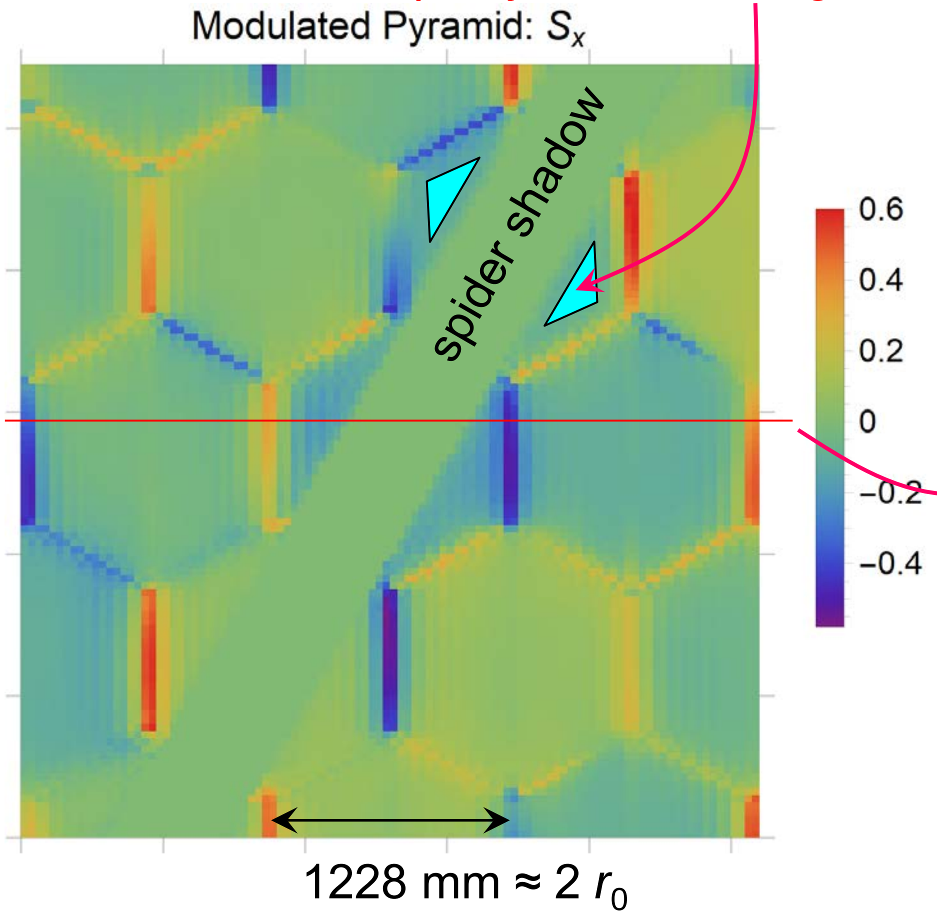


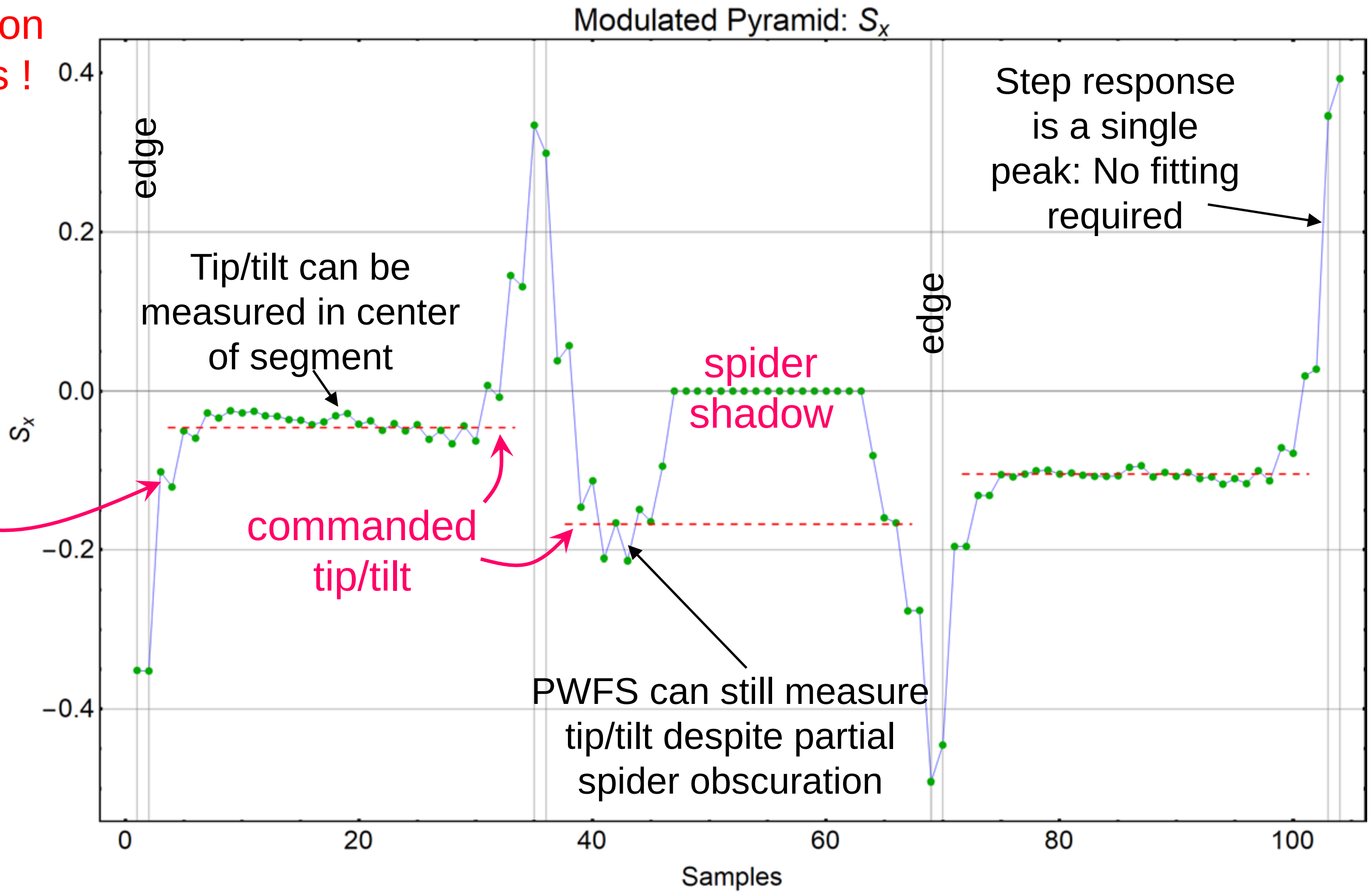


PWFS Slice Across Spider Shadow

1780 nm, r_0 : 602 mm (14.6 samples)

➔ Can sense tip/tilt even on partly obscured segments !





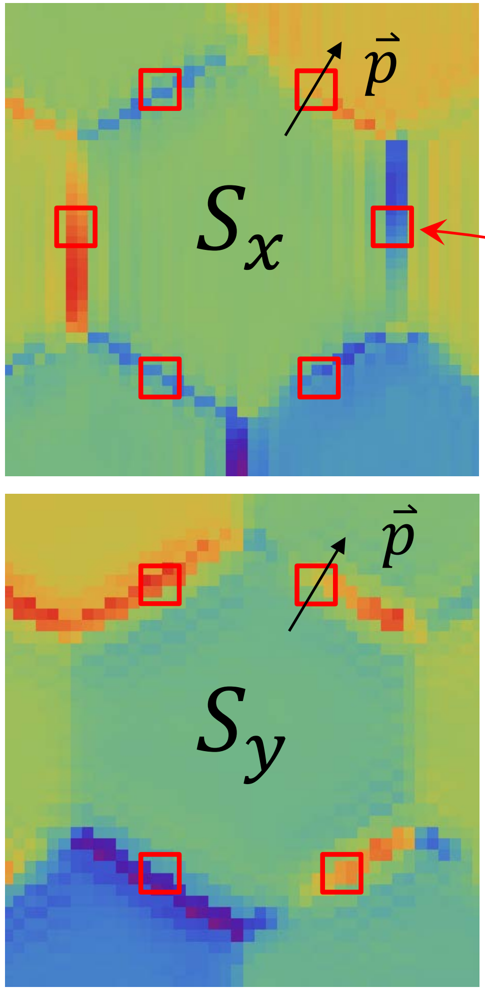
Step Reconstruction

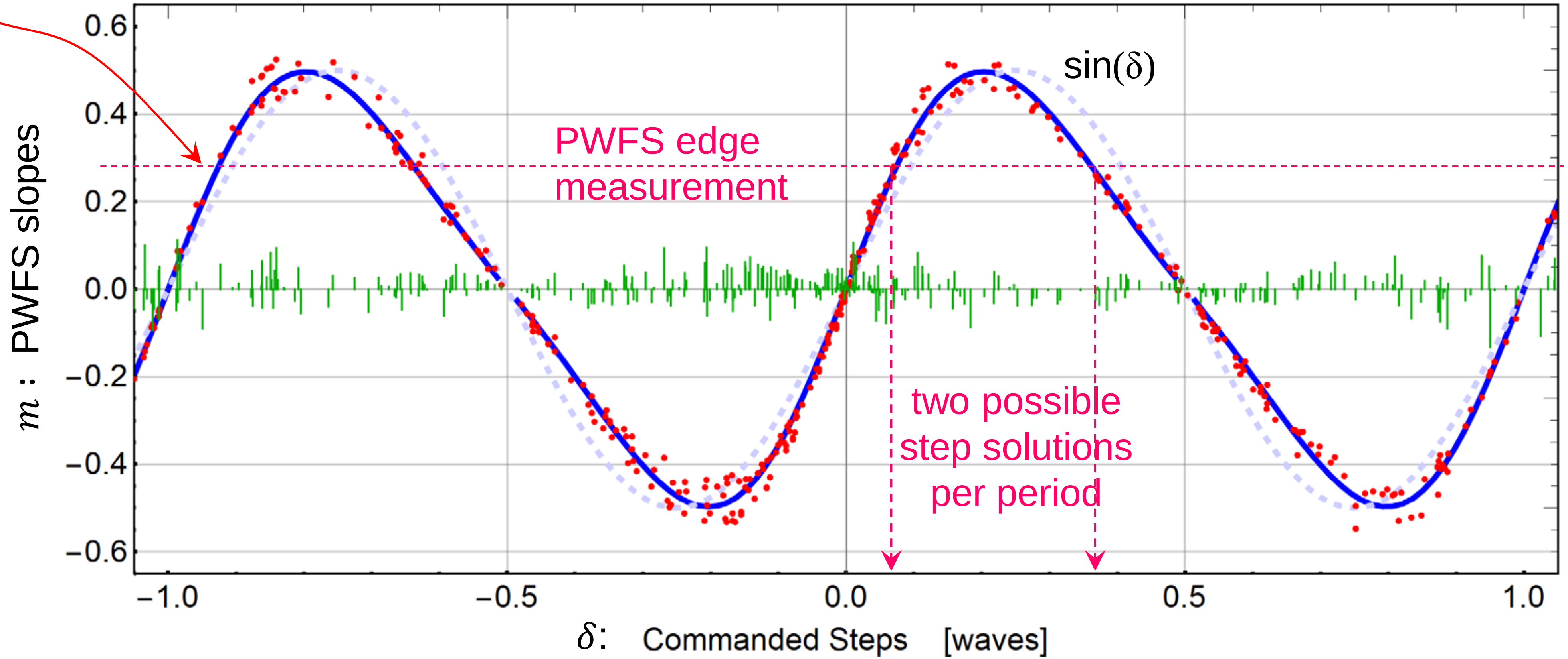
Fit function: skewed sine $m \coloneqq (\{S_x, S_y\} - \varepsilon\{R_x, R_y\}) \cdot \vec{p} = \frac{a \sin(2\pi\delta)}{1 - b\cos(2\pi\delta)}$

$\delta' \coloneqq \delta - [\delta]$: step in waves, mapped back to the basic period [−0.5, 0.5]

Small correction in m for tip/tilt $\{R_x, R_y\}$ of the two adjacent segments

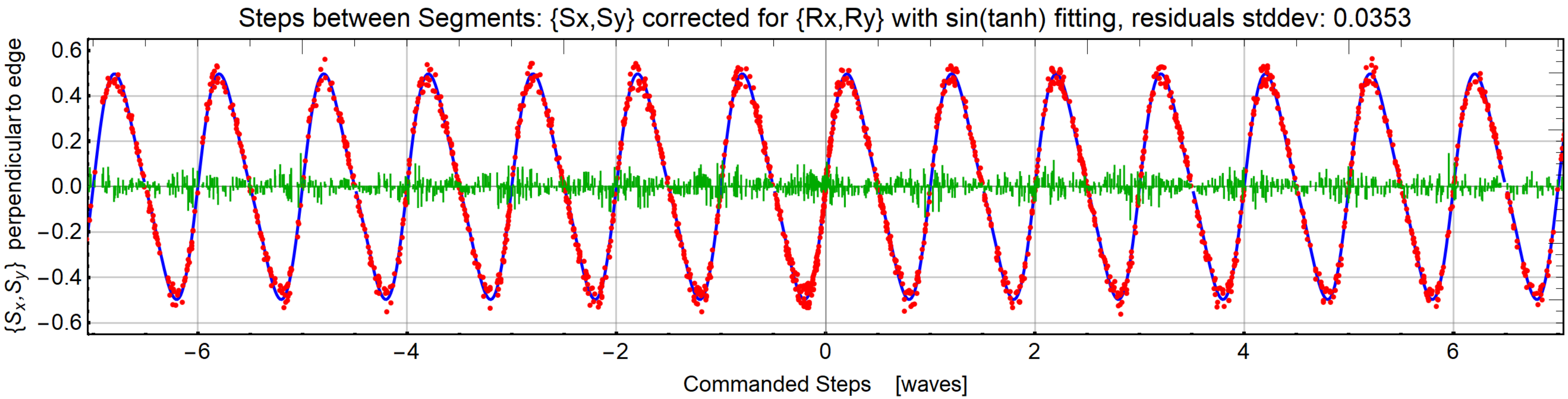
Parameter b to model residual saturation; smaller with larger modulation



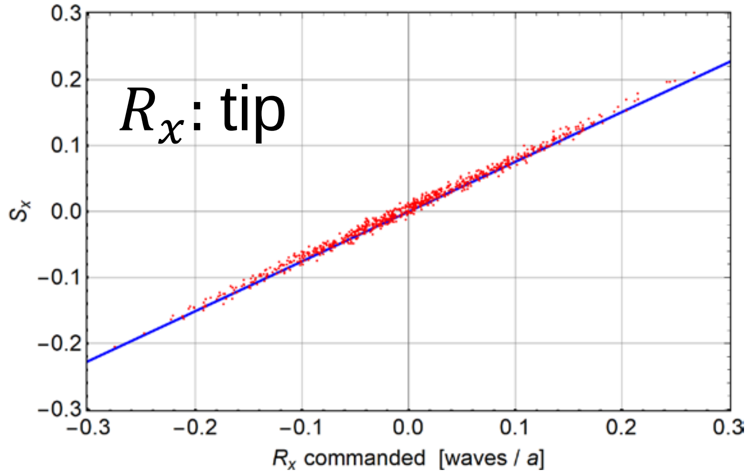


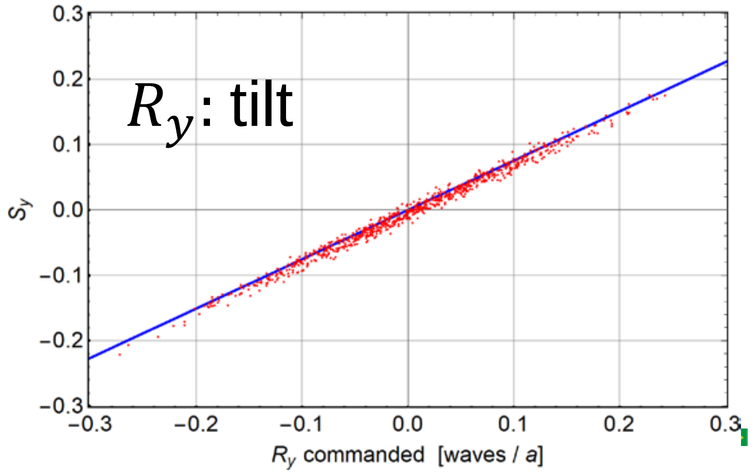
Fit Models for Steps and Tip/Tilt

Skewed sine is indeed a good fit function over many waves:

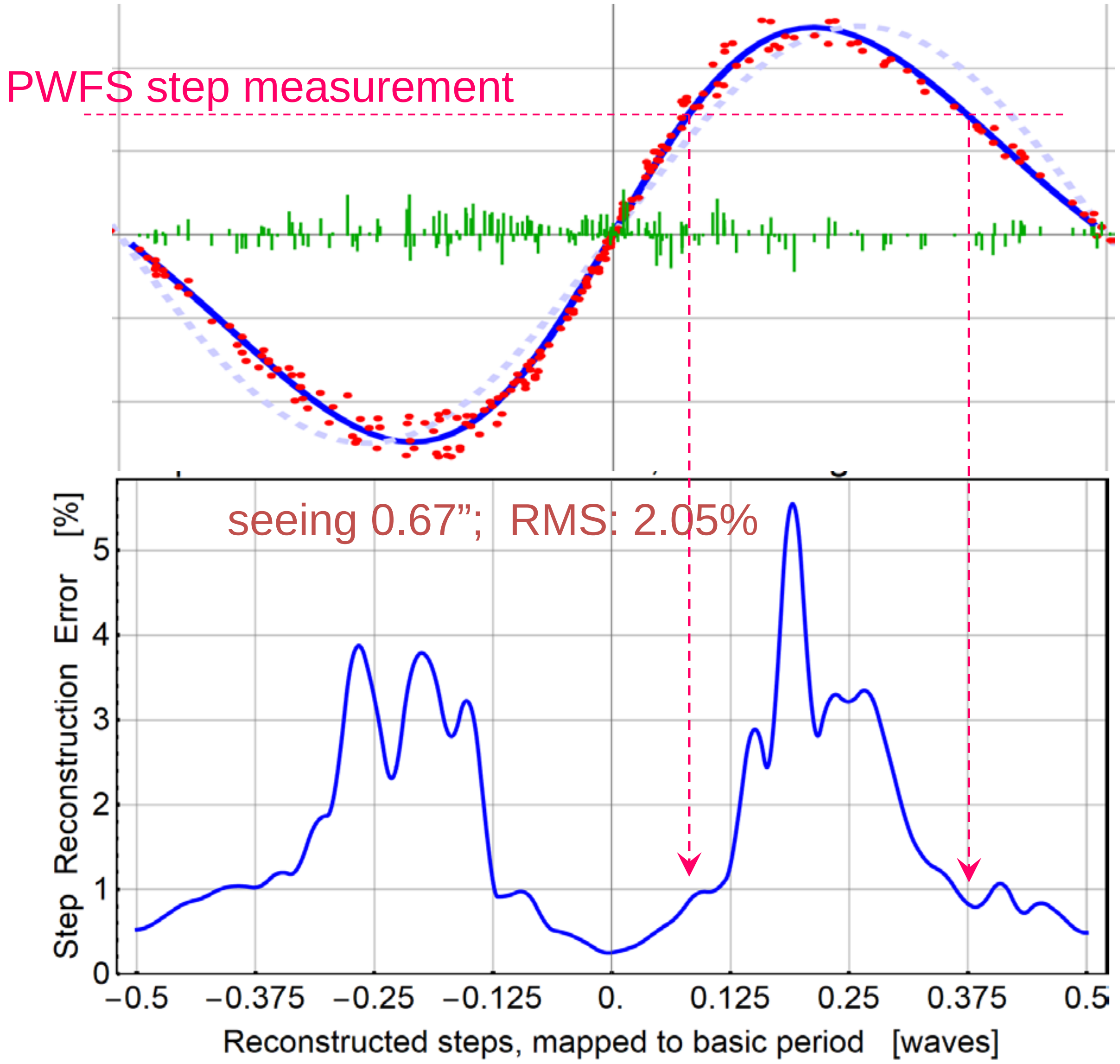


Random pistons picked from Gaussian PDF so that the steps have 6.3 waves OPD standard deviation; tip/tilt smaller by a factor 50

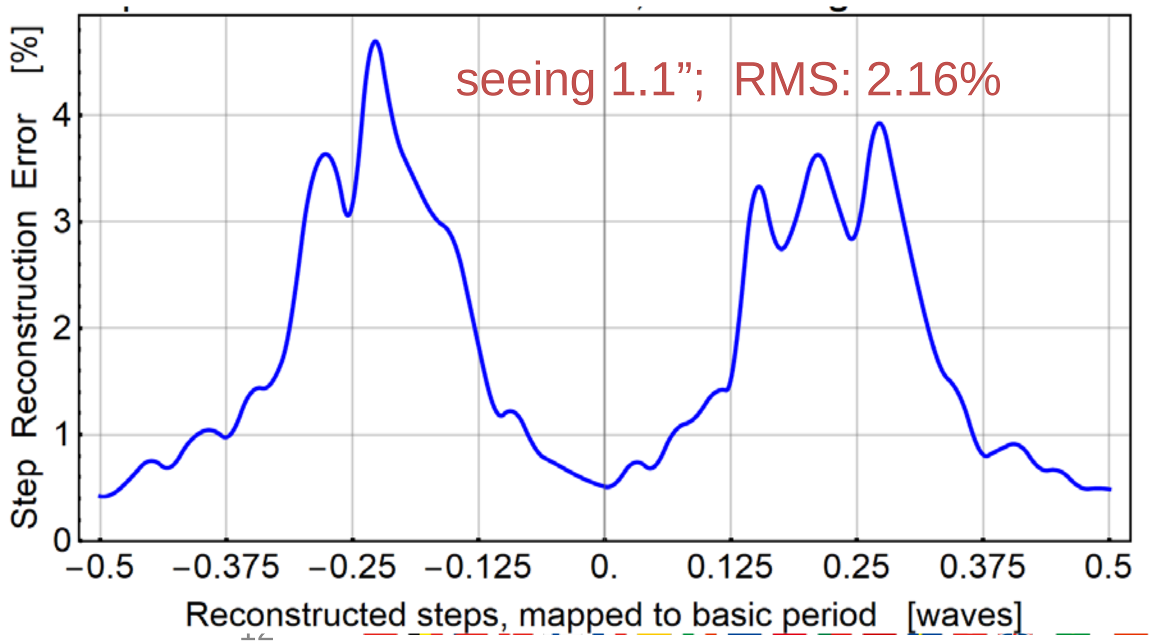




PWFS Step Inversion Error



➔ Step inversion error is highest near the peaks of the skewed sine and lowest near the zeros. We use this information in the multicolor step reconstruction and later in the Generalized Least Squares method to find most likely sol’s



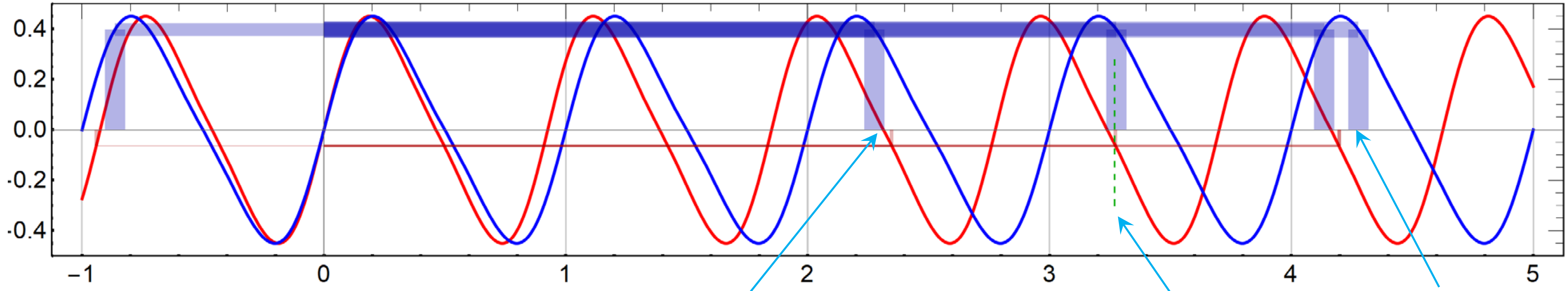
Maximum Likelihood Phasing Approach

- ELT M1 has 798 segments with 2262 inner edges (TMT: 492 / 1386)
- Each segment has 3 DoF: {piston,tip,tilt}. But we sense steps and tip,tilt
- Multi-color measurement to overcome phase ambiguity (synthetic wavelength) *
- Must be based on WFS error model (function of phase)
- Adopt a multilevel approach, using multicolor PWFS measurements:
  1. Sense $\{S_x,S_y\}$ near the segment centers (tip/tilt) and across edges (steps) for each color
  2. Compile list of possible {step,likelihood} pairs, using accurately calibrated WFS model
  3. Evaluate likelihood of triples of steps around each inner vertex (geometric consistency)
     → two ranked lists of possible steps per edge; pick matching step solution
  4. Reconstruction: Find state vector x that minimizes $(r\, \boldsymbol{V}^{-1} r)$ with $r = \boldsymbol{A} \cdot x - b$ (GLS)
- Steps 2 and 3 are key (algorithm not limited to a specific WFS)

* Esposito & Devaney, Beyond Conv. AO (2001)

Multi-Color Step Solution Overlap

Likelihood of correct step solution is given by the solution overlap:



spurious step solution (bad overlap)

error bars

correct step solution: good overlap

Assuming the individual color step measurements have Gaussian distributed errors (with different errors σ_i), integrate the product of the probability of several colors: This is the likelihood that the solutions pertain to the correct step

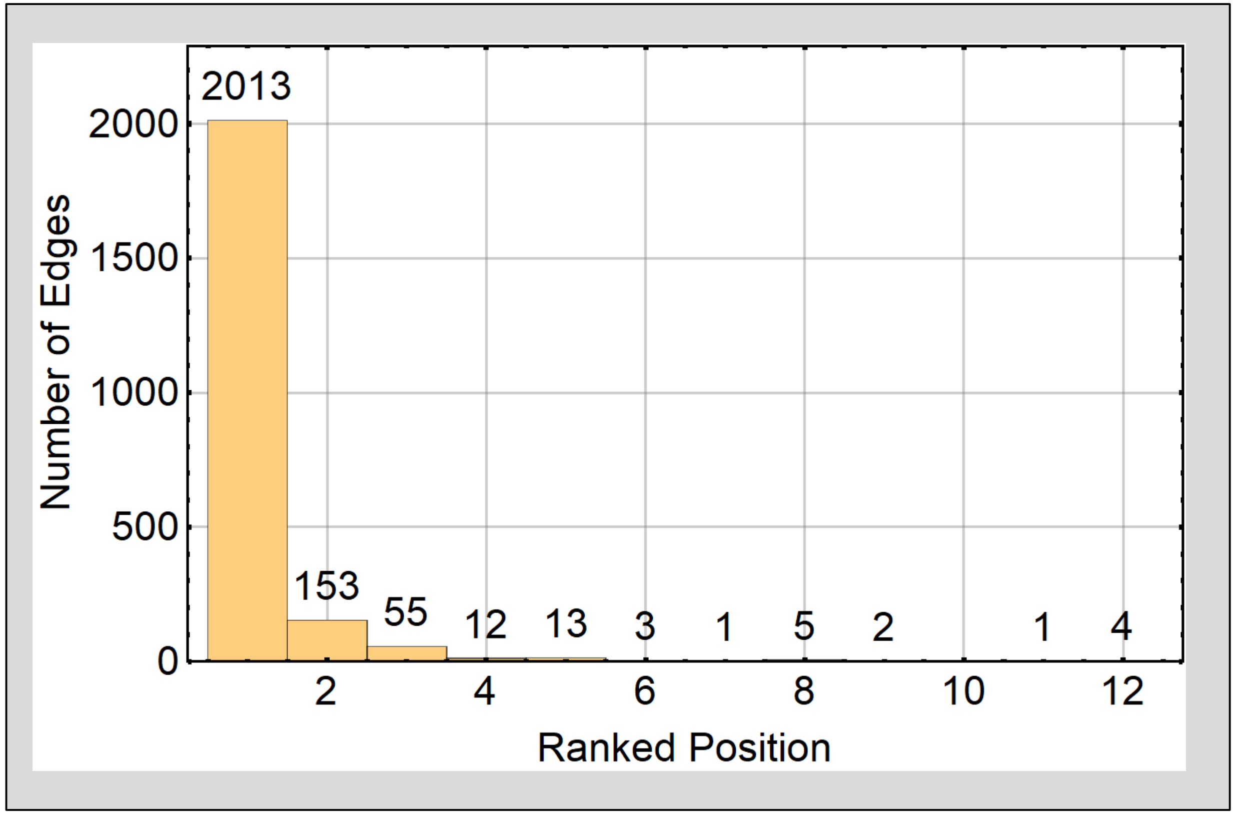
$$P_2 = \exp\left(-\frac{(x_1 - x_2)^2}{2(\sigma_1^2 + \sigma_2^2)}\right),$$

$$P_3 = \exp\left(-\frac{\sigma_1^2(x_2 - x_3)^2 + \sigma_2^2(x_1 - x_3)^2 + \sigma_3^2(x_1 - x_2)^2}{2(\sigma_1^2\sigma_2^2 + \sigma_3^2\sigma_2^2 + \sigma_1^2\sigma_3^2)}\right)$$

$$P_4 = \ ...$$

number of colors

Step Solutions vs. Likelihood



Histogram showing the positions of the *correct* step solutions in a ranked list (2262 steps, 4 colors, 12 solutions per step in the list)

➔ Fraction of correct solutions in first position (90% in this example) is a strong function of the step inversion error and number of colors

➔ Further elimination of spurious solutions needed...

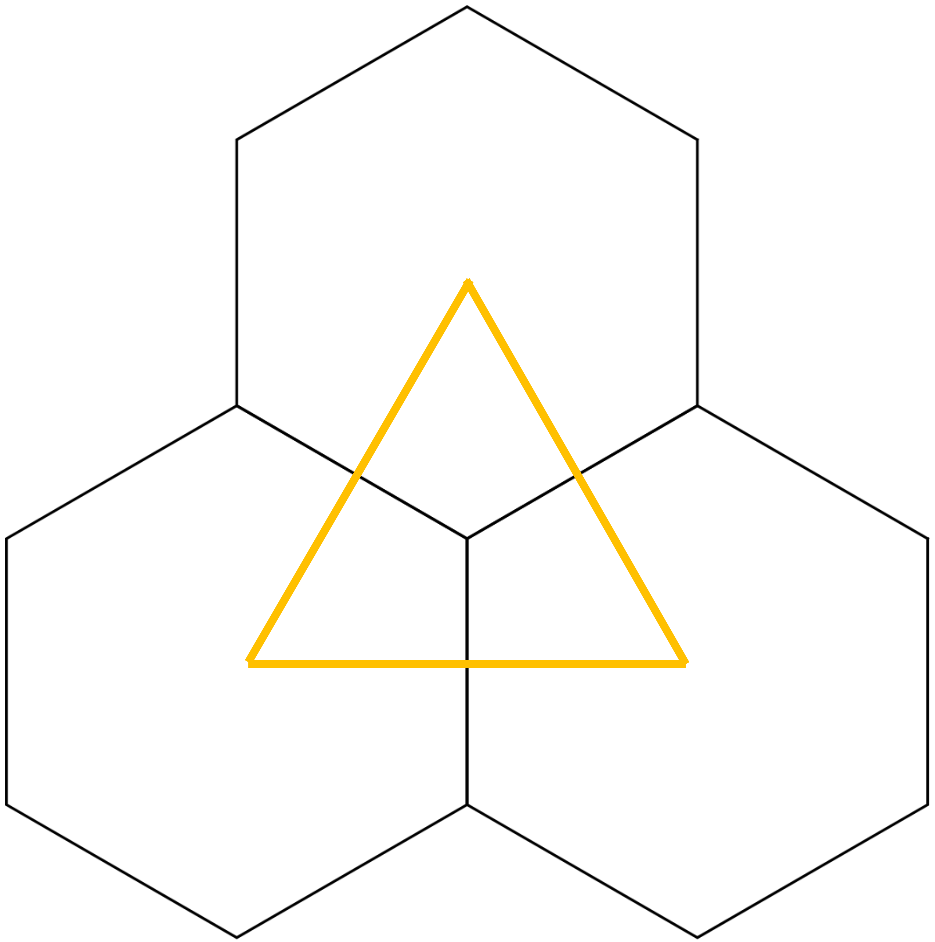
Exploit Geometric Redundancy on Clovers

- Clover: Set of three segments sharing a vertex
- Follow the yellow triangle: The sum of directed {tip,tilt} and steps must be zero (“phase closure”)
- ***Step 3***: Set up list of step triples, compute the geometric error Δz in the directed loop sum
- Rank triples by combined likelihood:

$$P_{\text{clover}} = P_{\text{multicolor overlap}} \times P(\Delta z)$$

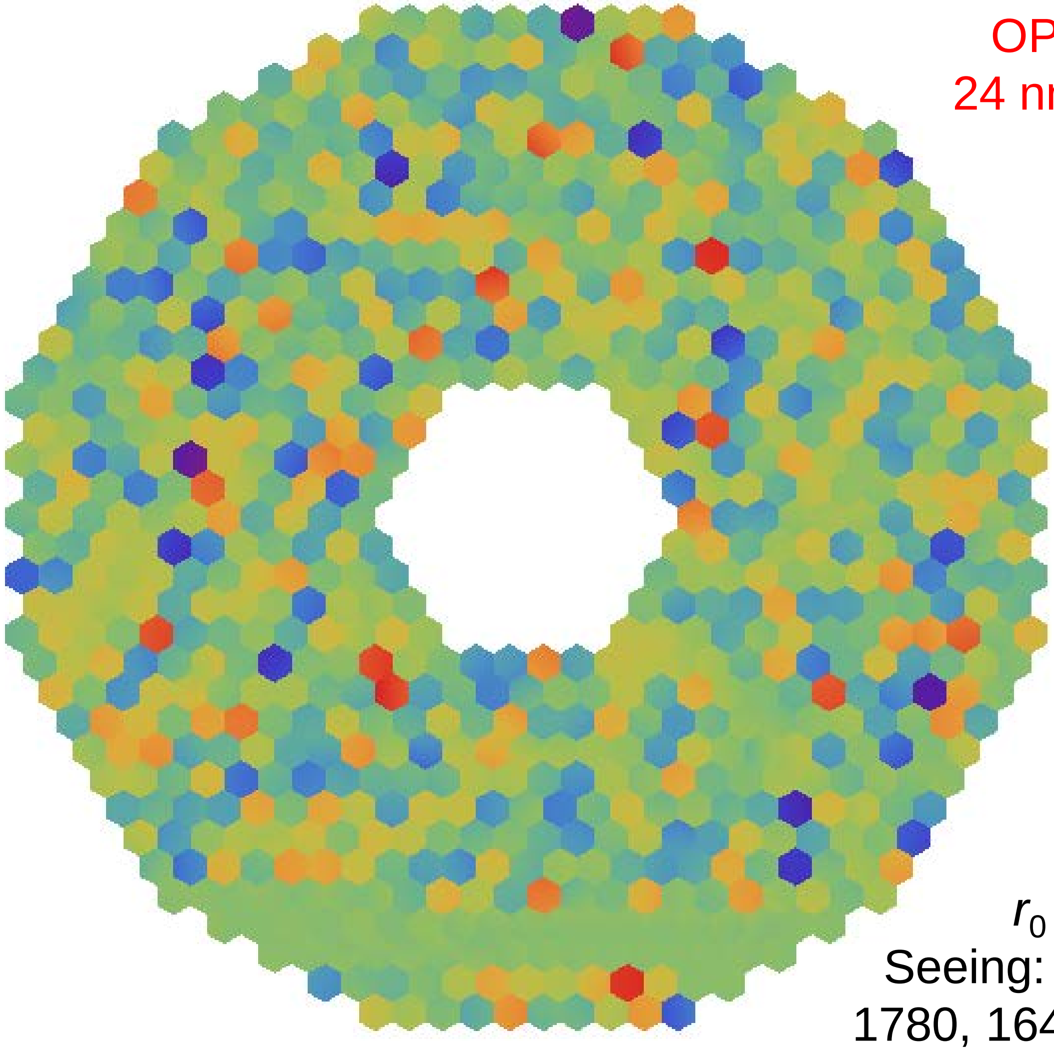
➔ Finally, compare the ranked triples for each edge between its two enclosing clovers and select most likely match(es)

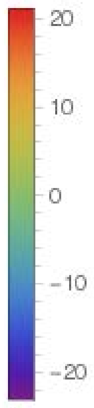
- Iterate with GLS; use large variances for high residual



H/J-Band Reconstruction (with spiders)

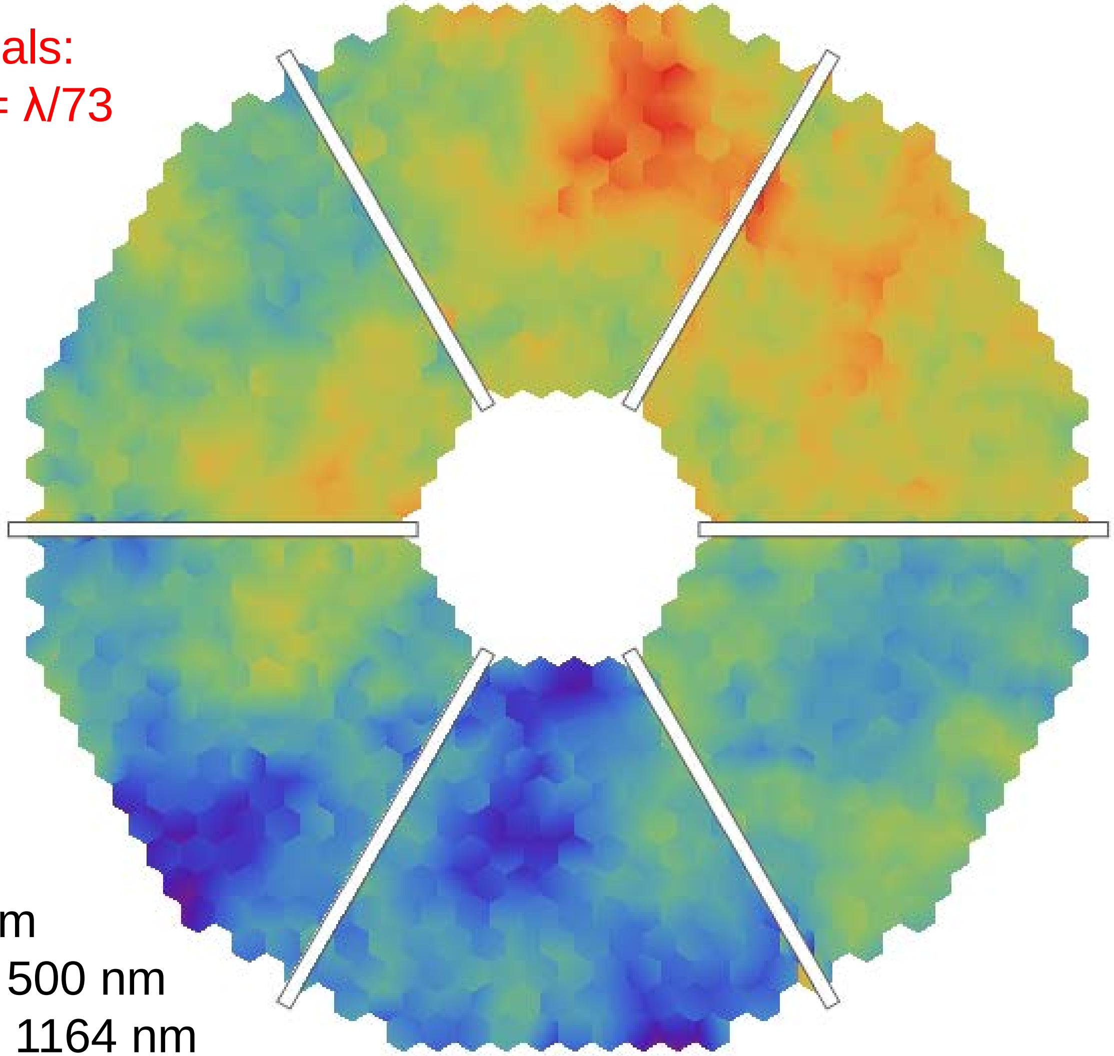
Before Phasing





Post-Phasing Residuals

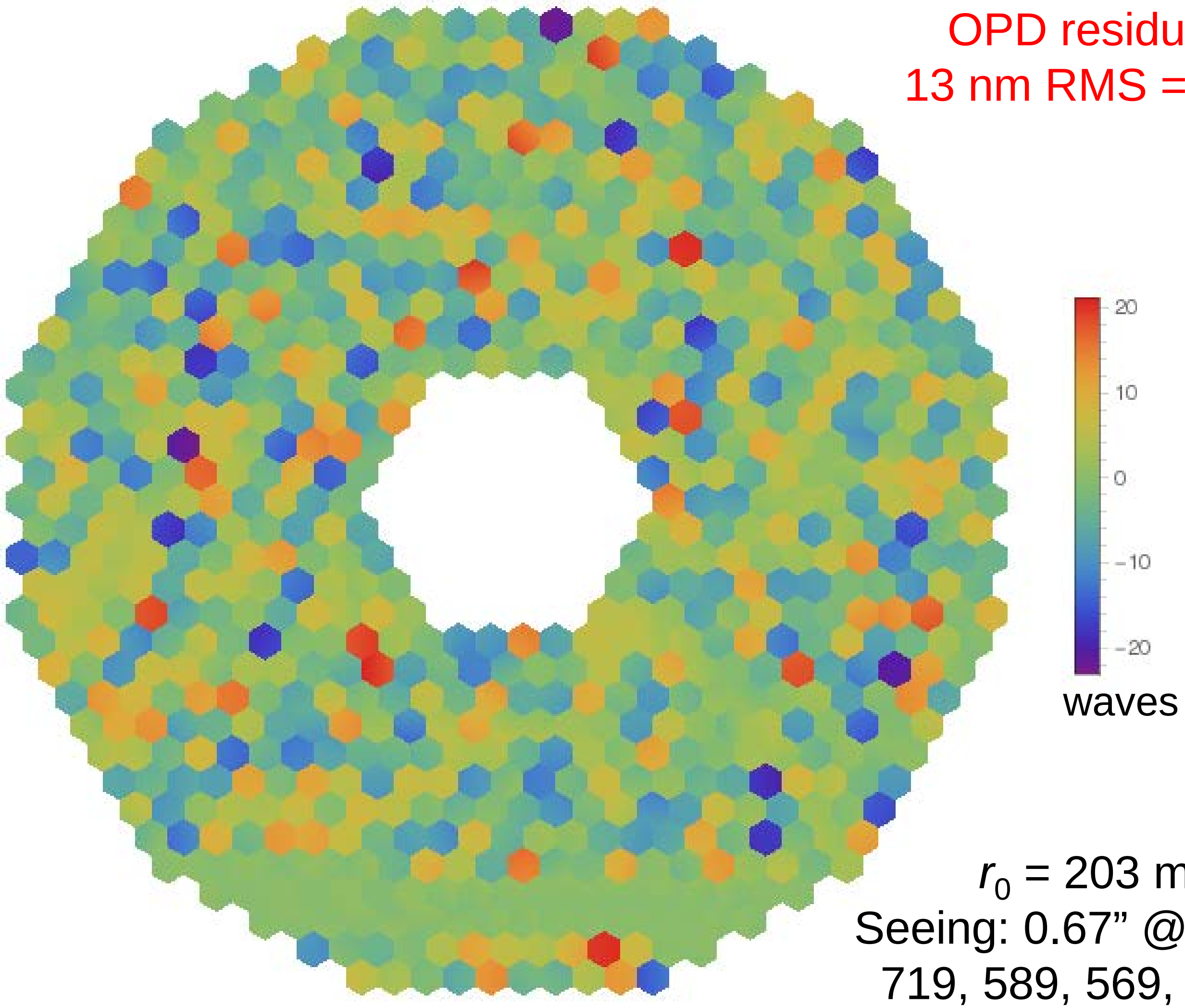
OPD residuals:
24 nm RMS = λ/73



r_0 = 602 mm
Seeing: 0.67” @ 500 nm
1780, 1647, 1477, 1164 nm

R/V-Band Reconstruction (with spiders)

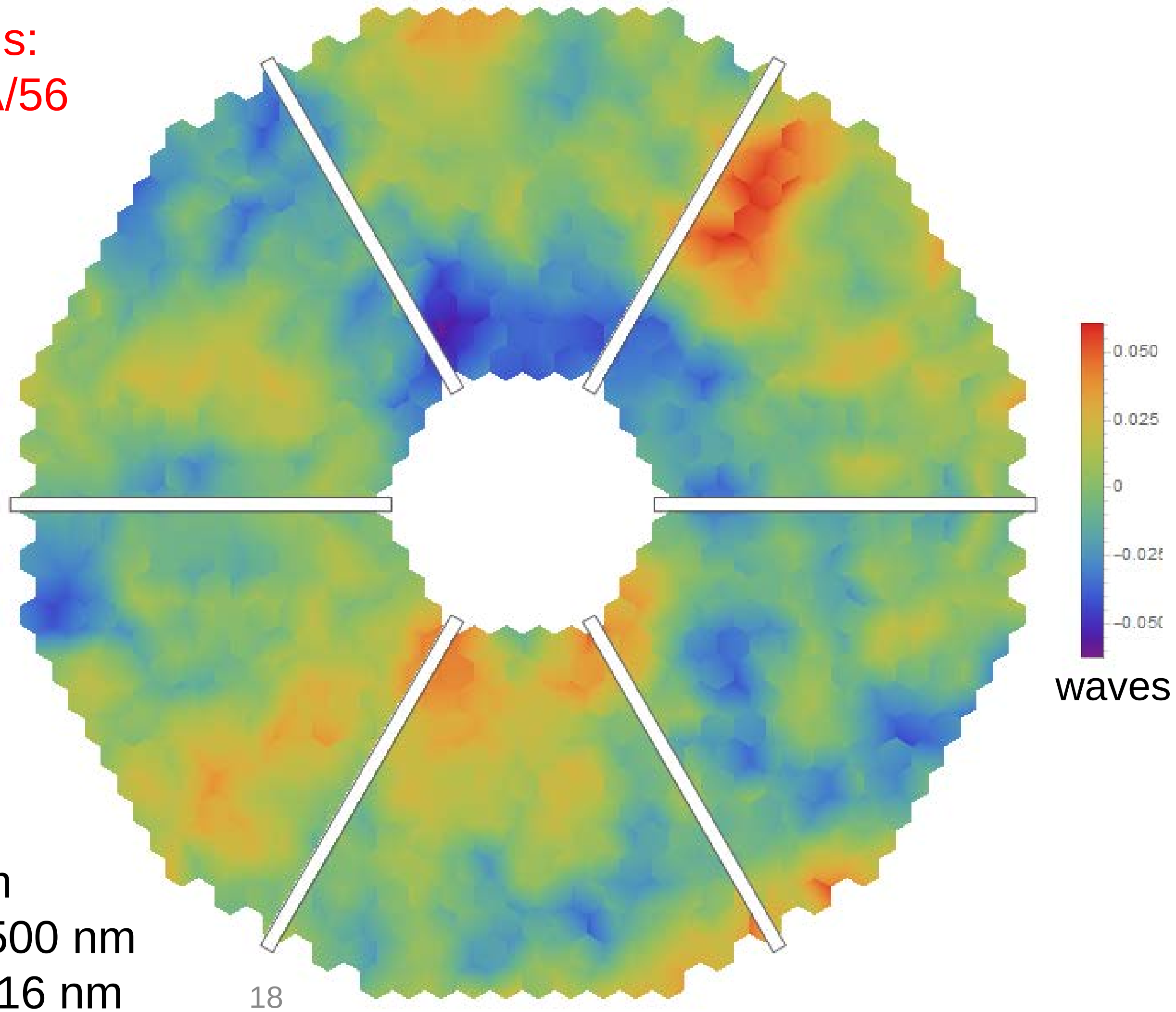
Before Phasing



OPD residuals:
13 nm RMS = λ/56

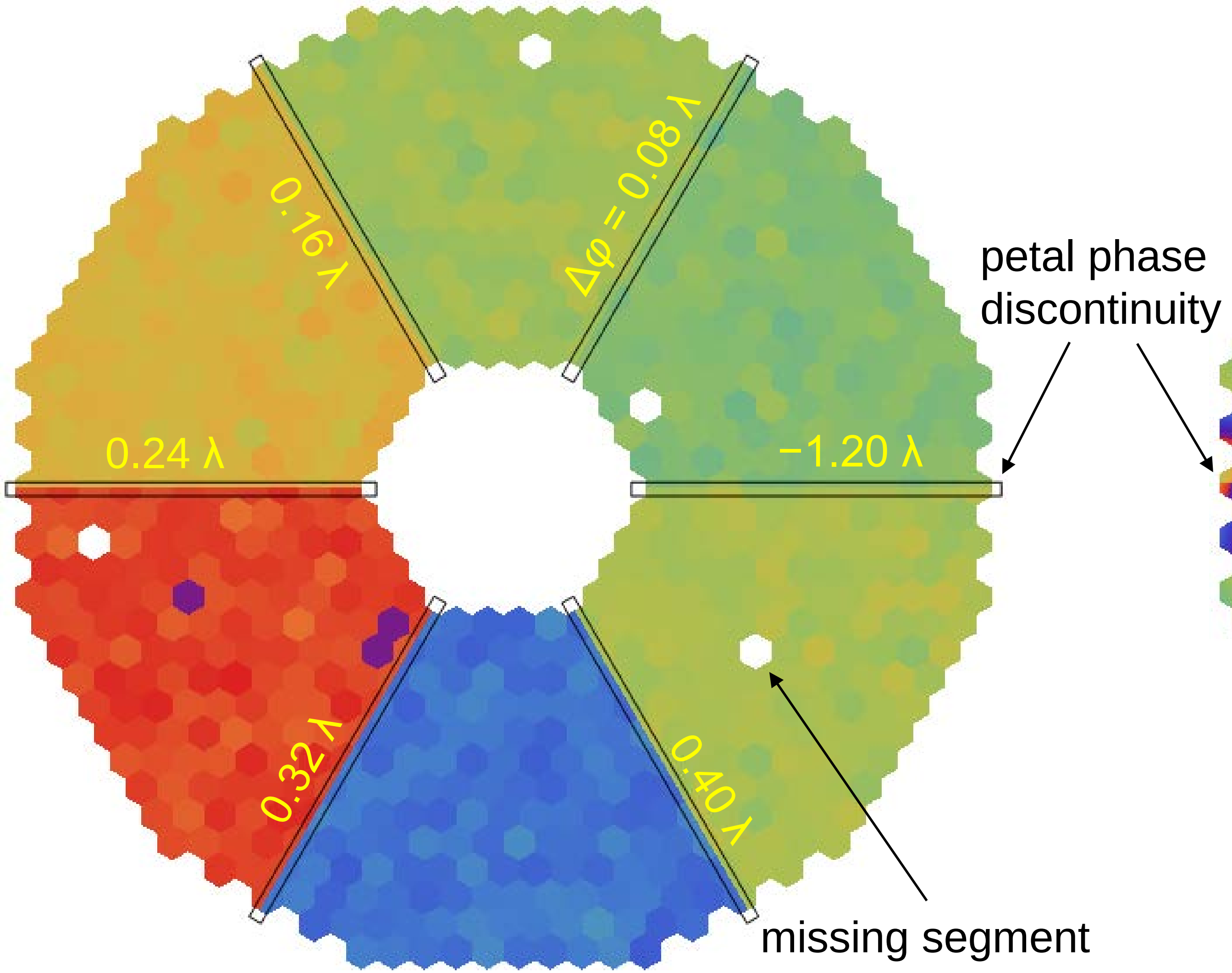
r_0 = 203 mm
Seeing: 0.67" @ 500 nm
719, 589, 569, 516 nm

Post-Phasing Residuals

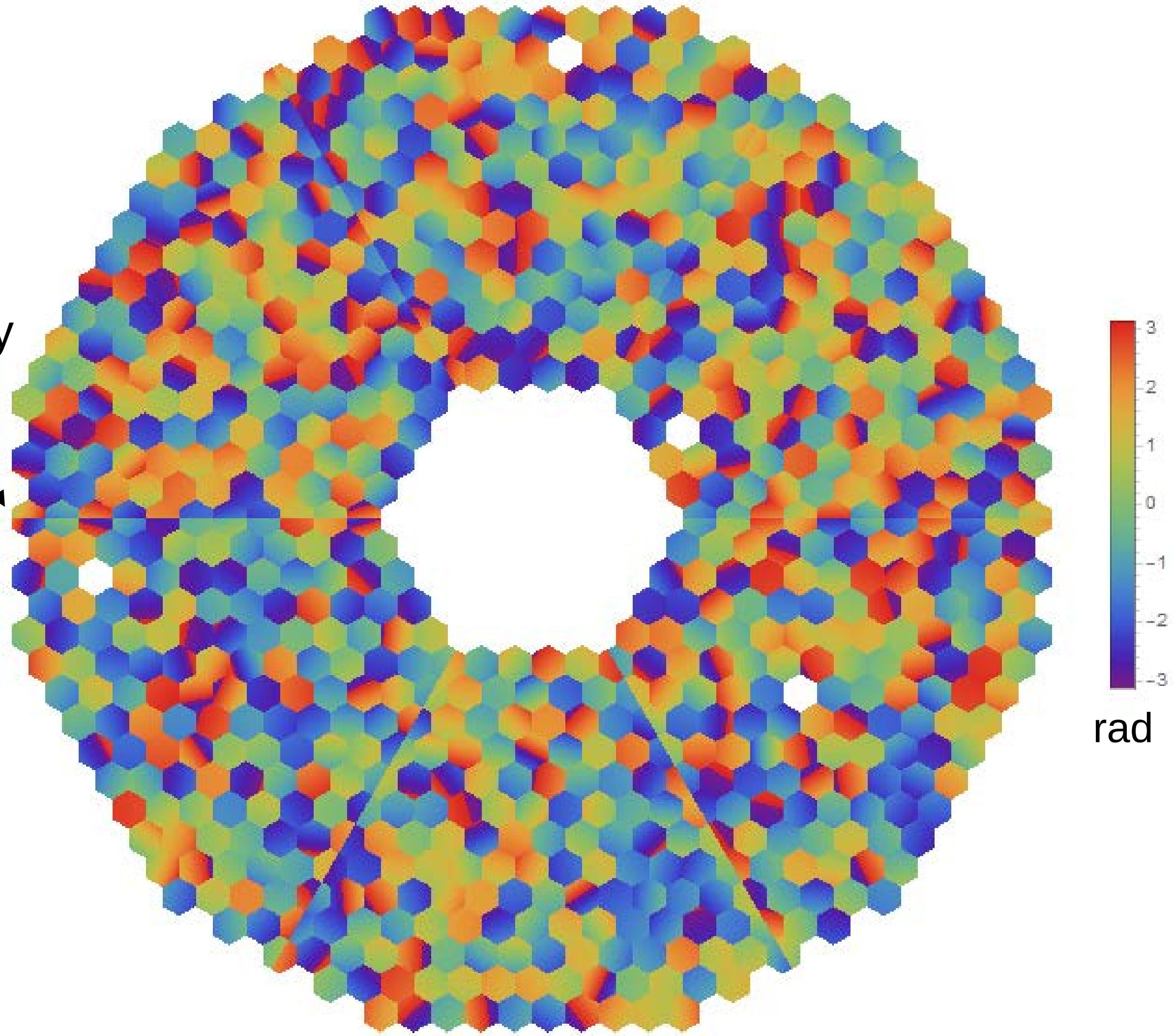


Add Petal Phase Offsets + Missing Segments

Petal phase offsets

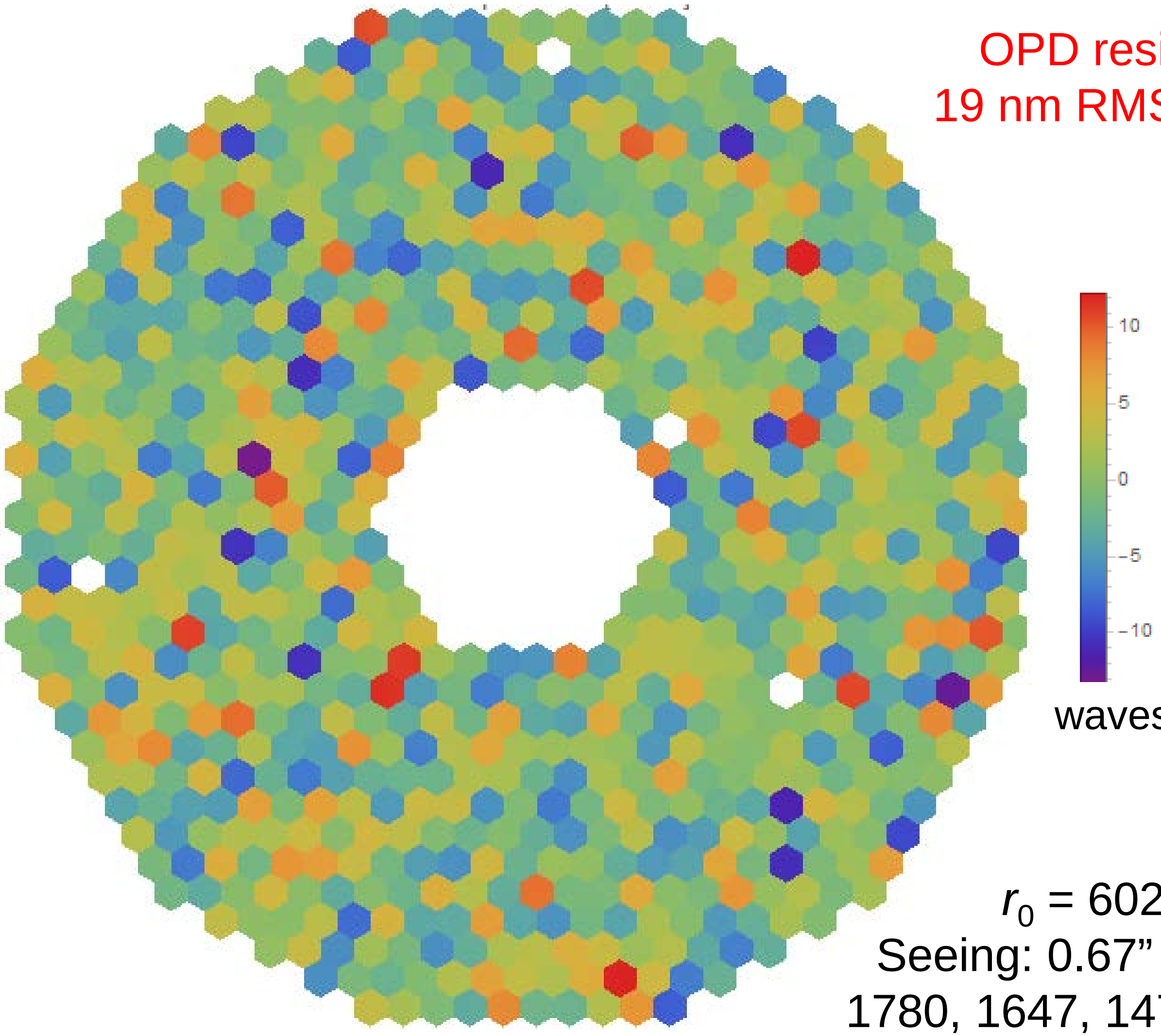


Phase in the pupil plane

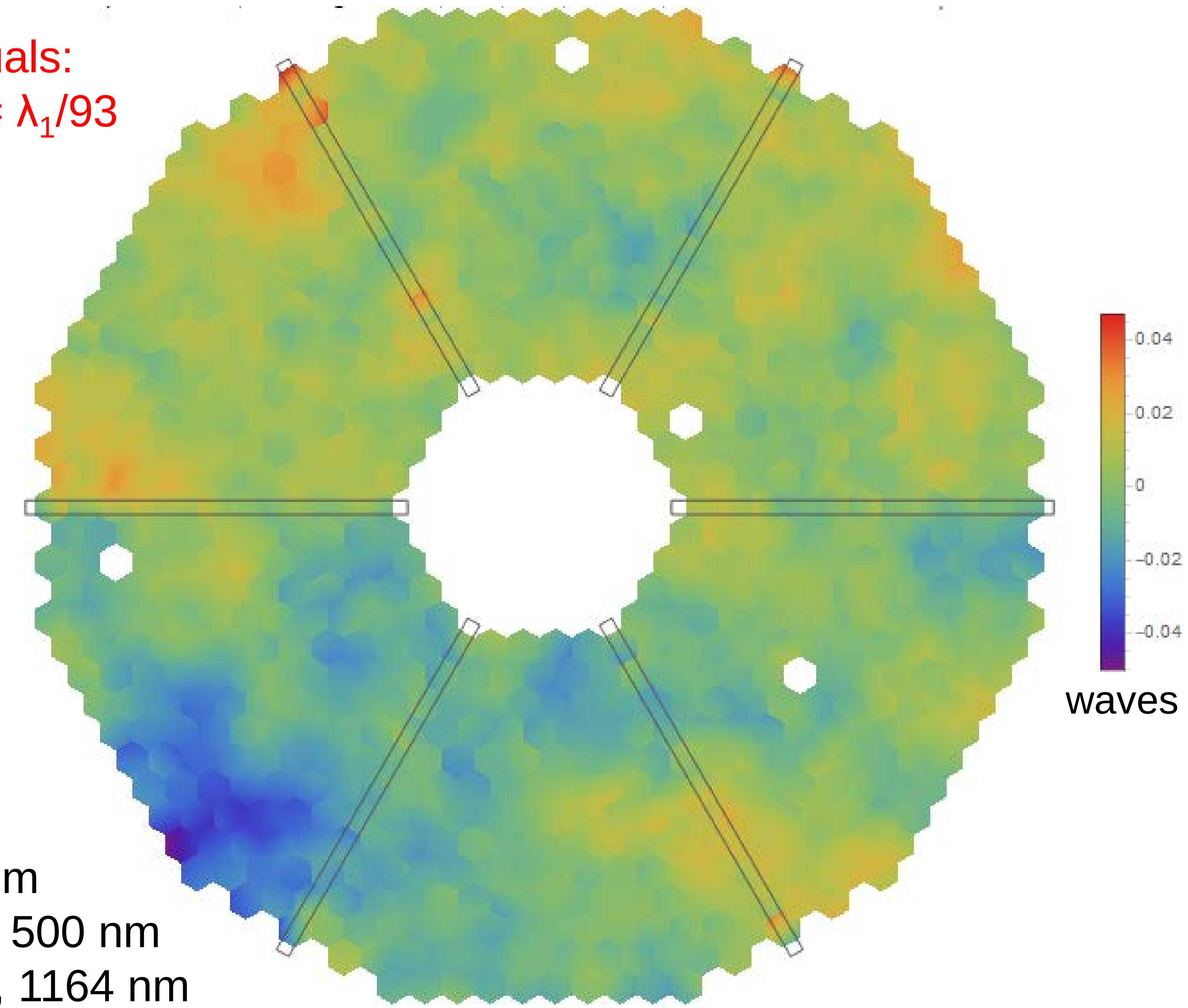


H/J-Band: Missing Segments + Petaling

Before Phasing



Phasing Residuals

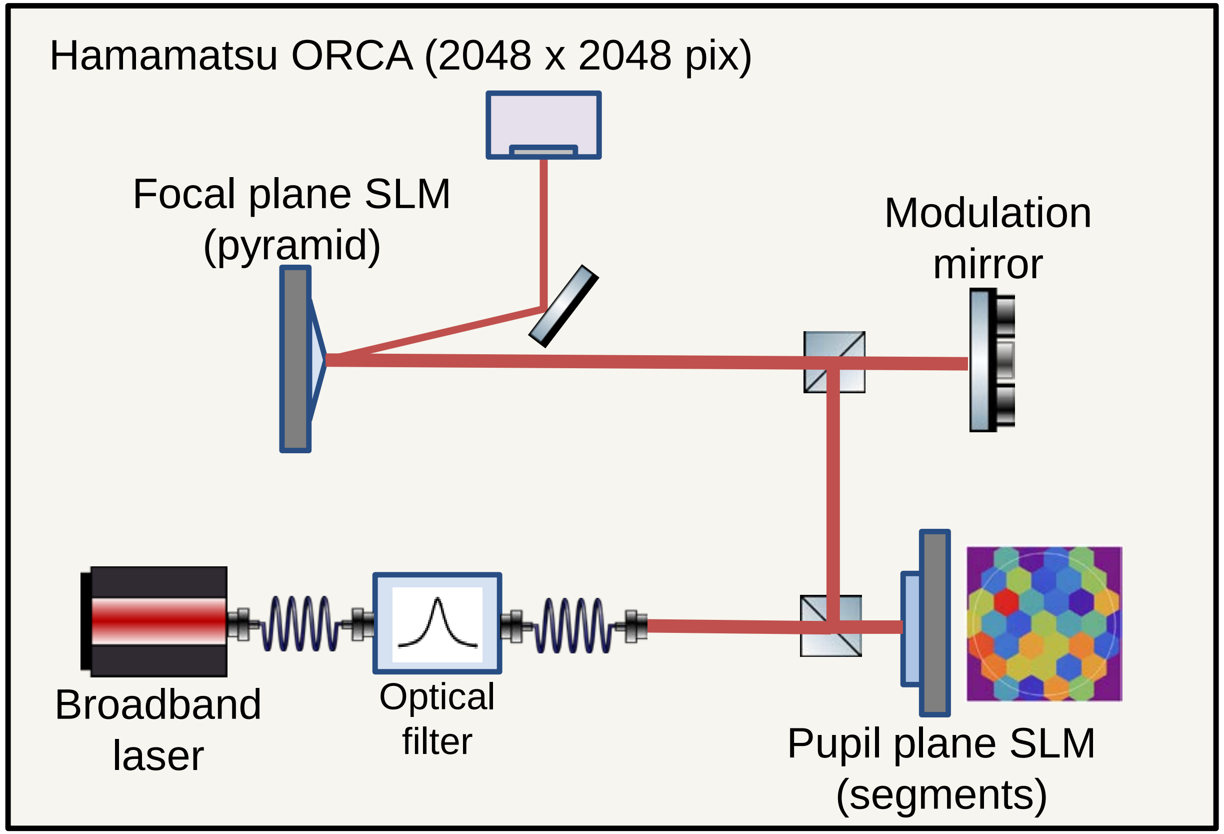


OPD residuals:
19 nm RMS = $\lambda_1/93$

r_0 = 602 mm
Seeing: 0.67” @ 500 nm
1780, 1647, 1477, 1164 nm

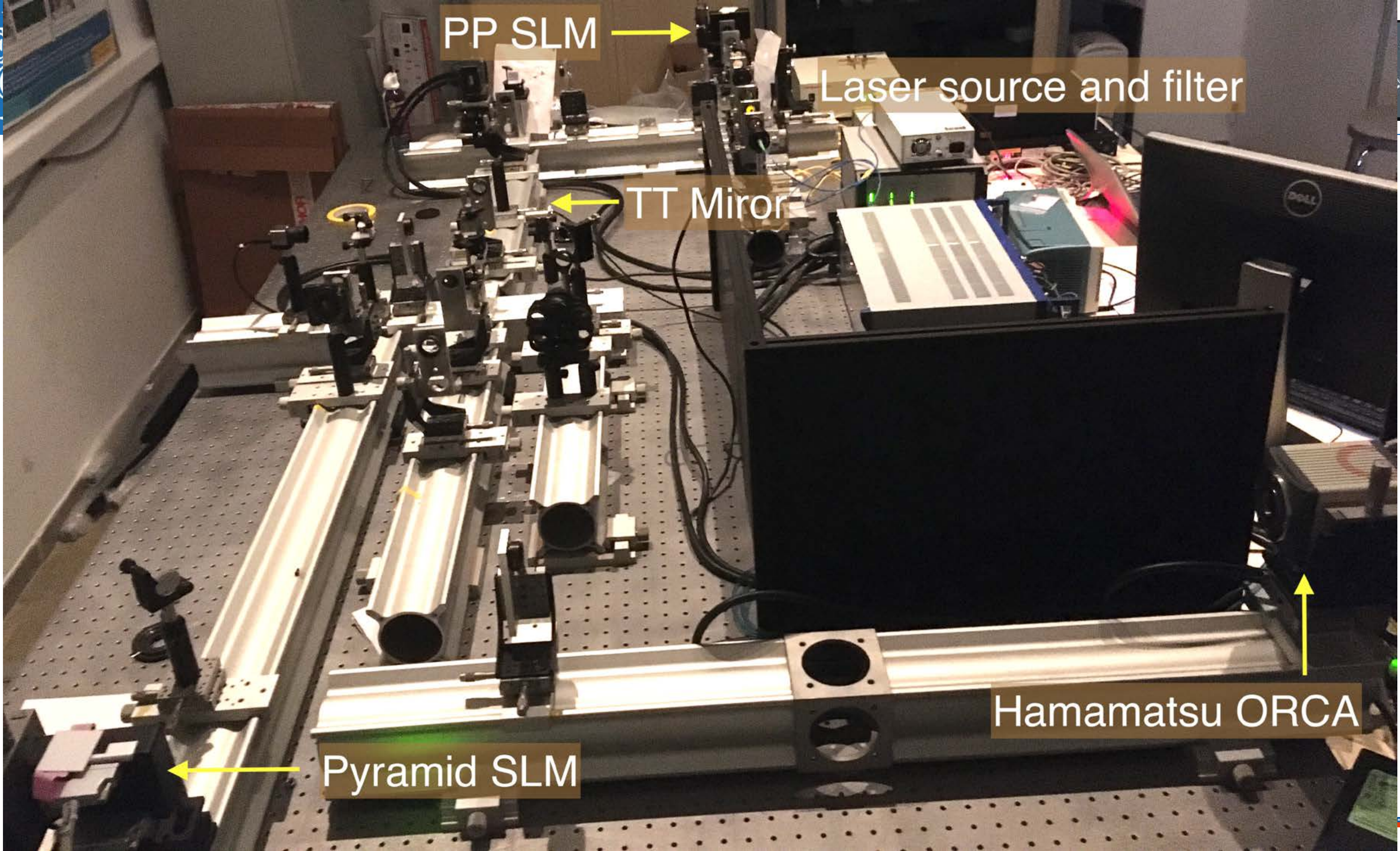
LOOPS Experiments !

LOOPS Phasing Setup



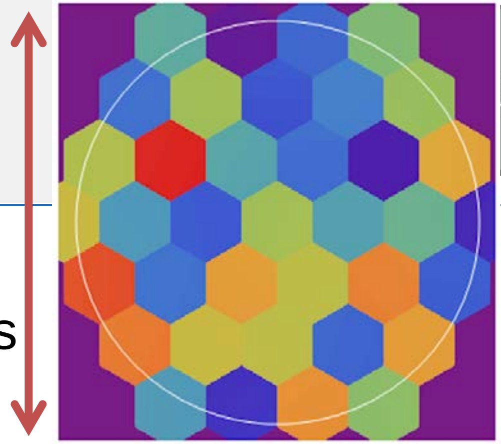
Schematic of setup @ LOOPS bench

- Pulsed supercontinuum laser source
- Power density ≈ 50 μw/nm
- Filtered with acousto-optical tunable filter
- ≈ 1 nm linewidth
- WF camera with 6 μm pixel size (compare with 24 μm of OCAM2)

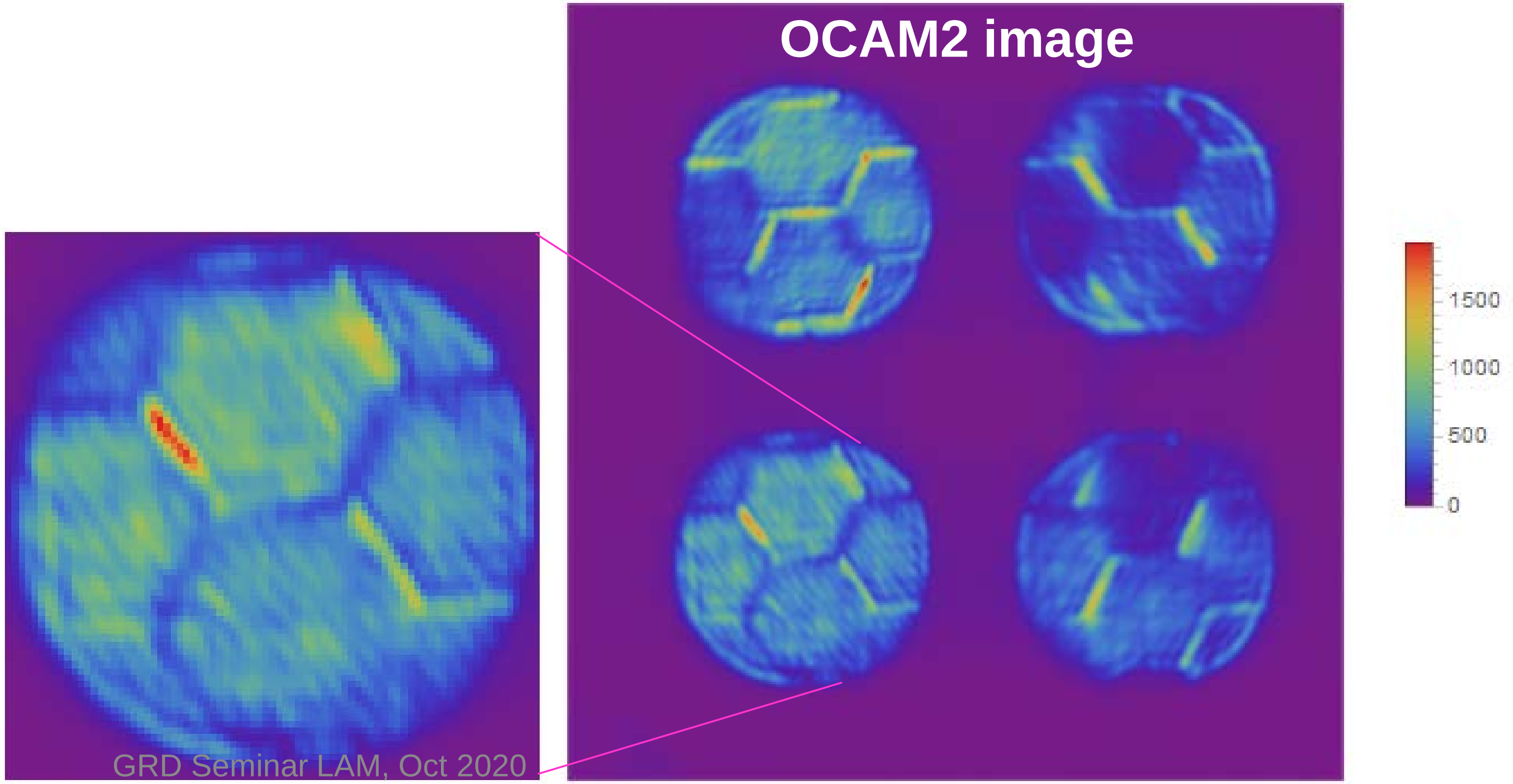

PP SLM
Laser source and filter
TT Miror
Hamamatsu ORCA
Pyramid SLM

First Measurements

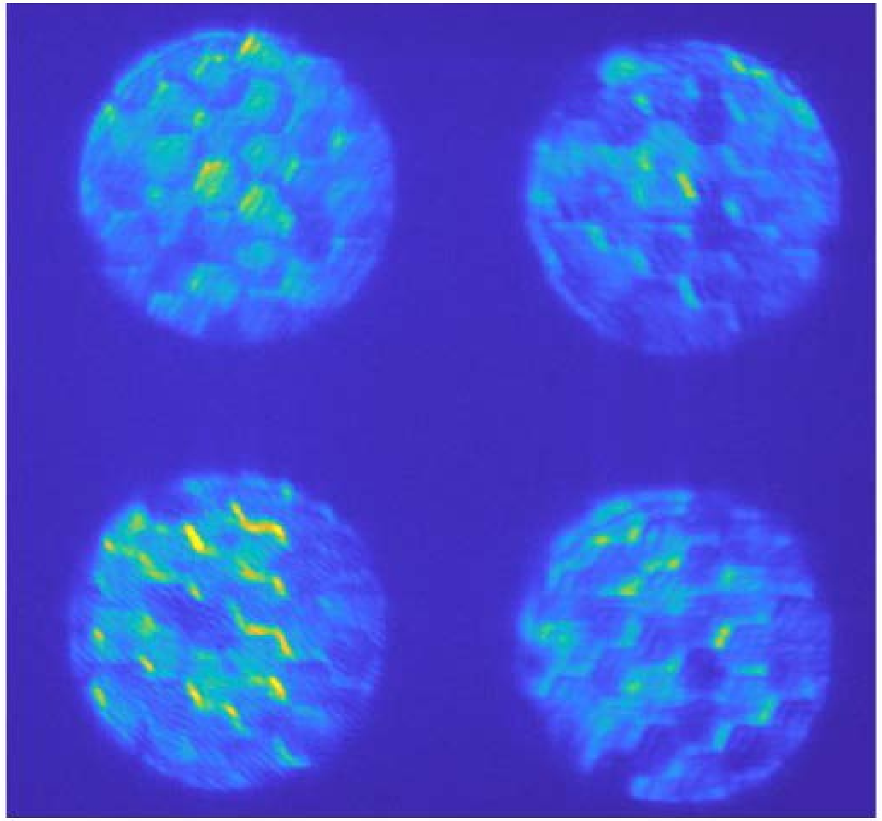
- First experiment with OCAM2 (80 pix/pupil , ~50 pix/segment)
- Switched to Orca: Same pupil footprint, but 4x resolution (318 pix/pupil, ~50 pix/segment)
- Emulating 37 segments (Keck, GTC mirror pattern)



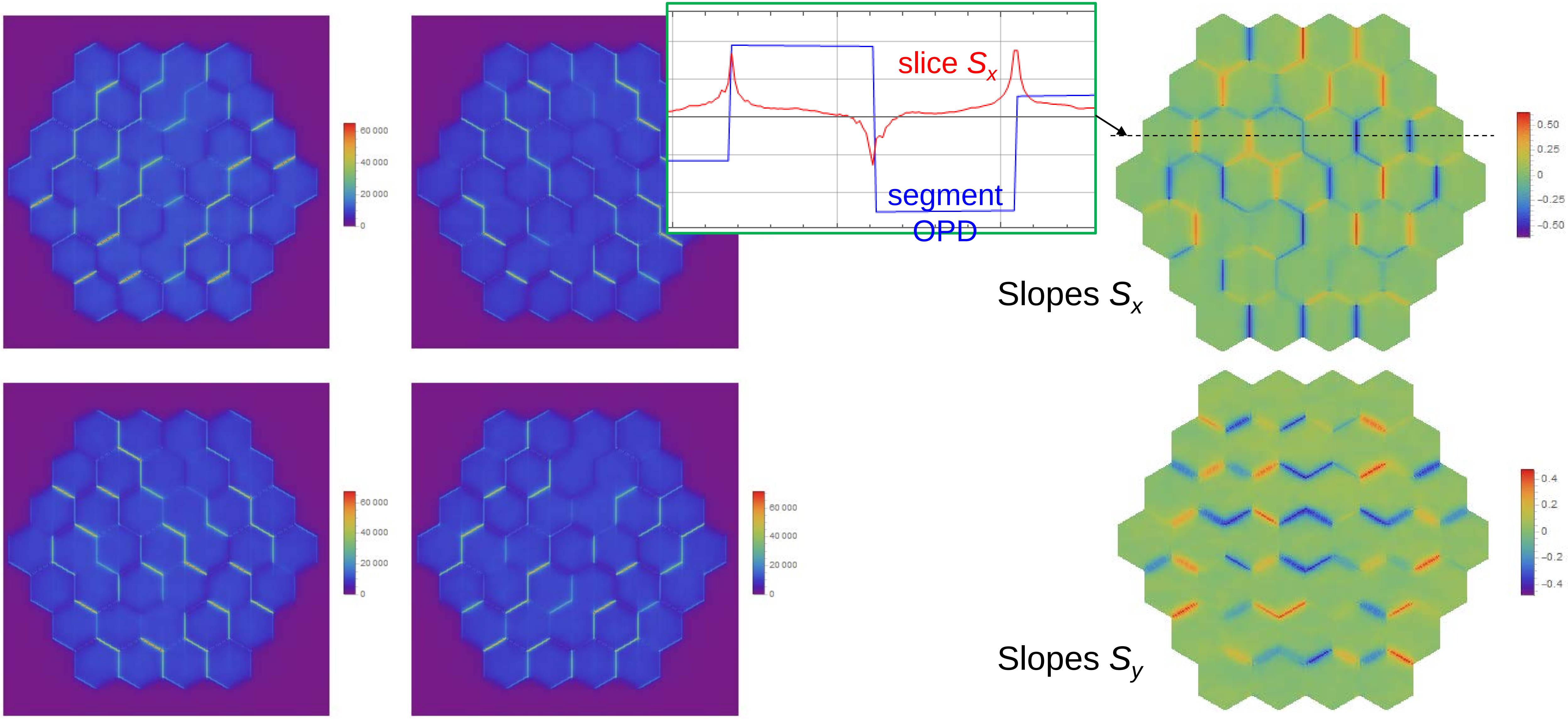
320 pixels



Orca image (<u>preliminary</u>)



Simulation…



60 000
40 000
20 000
0
slice S_x
segment OPD
Slopes S_x
0.50
0.25
0
−0.25
−0.50
Slopes S_y
0.4
0.2
0
−0.2
−0.4

Conclusions

- Primary mirror phasing becomes more demanding in ELTs
  - Ability to quickly/frequently phase M1 would be a valuable asset
  - Desirable to sense both, segment tip/tilt and steps, in parallel
  - Response function, cross-talk, linearity vary with WFS type
  - Segment registration and reconstruction algorithm must be optimized for WFS type to get best performance. Room for performance increase!?
- Numerical demonstration of “one shot” multicolor segment phasing with PWFS in the low-noise limit, both in R/V and H/J bands
  - Works with spider obscuration, missing segments and petaling
  - To be done: model detector/sky noise, radial segment-to-pupil compression, segment registration on skewed pixel grid, lower pixel count, op. scenarios
- Experiments on LOOPS bench @ LAM: First results look promising...

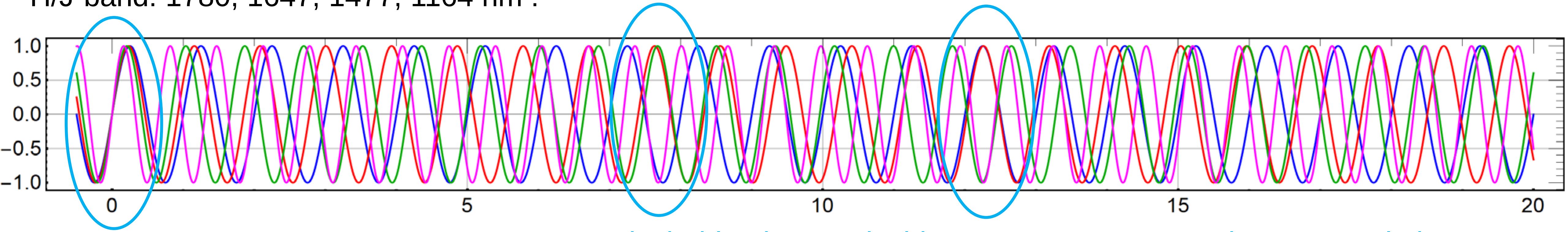
... Encore

Additional Material

Multicolor Sine Overlap

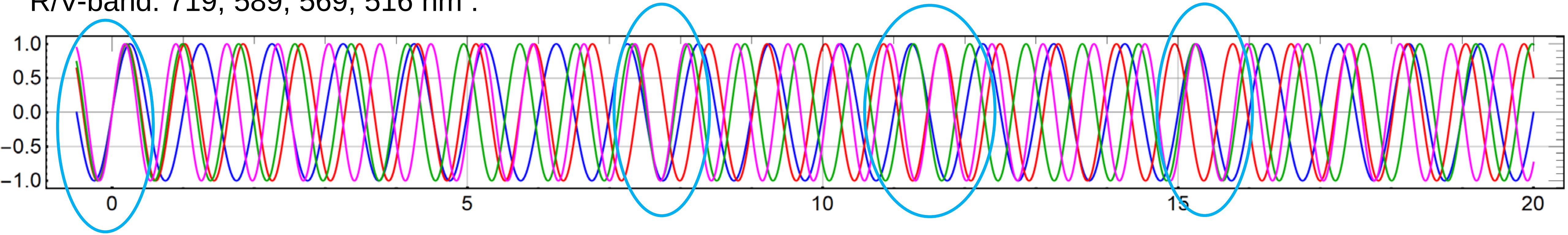
- Plot sine functions up to 20 waves and highlight areas where phases of 3 or even 4 colors agree (near peaks also phase +π agreement)

H/J-band: 1780, 1647, 1477, 1164 nm :



undesirable phase coincidences; may cause spurious step solutions

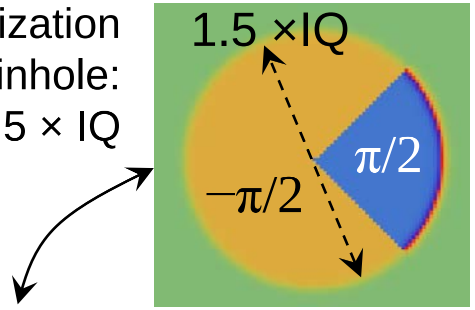
R/V-band: 719, 589, 569, 516 nm :

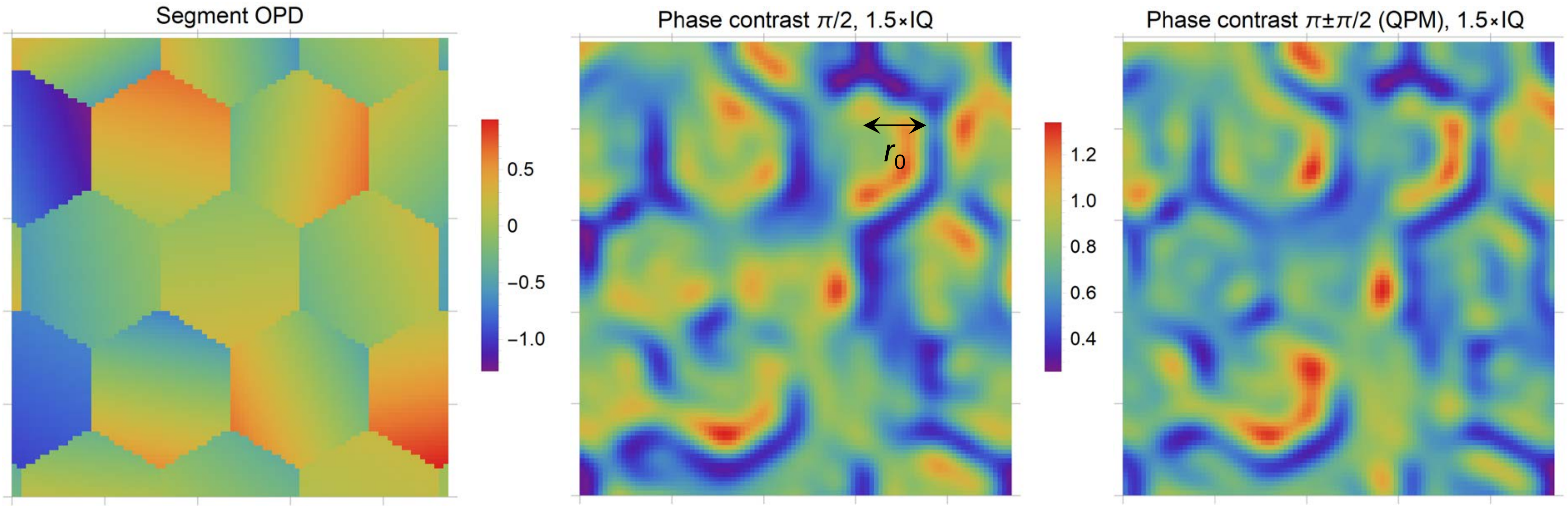


Phase Contrast WFS

- 1780 nm (H-band), averaged over 4000 phase screens
- Seeing: 0.67” (at 500 nm), IQ: 0.37”, r_0 : 602 mm (14.6 samples)

+ apodization pinhole:
Ø = 3.5 × IQ



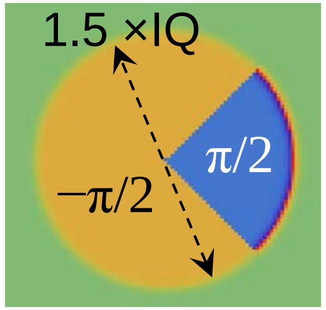


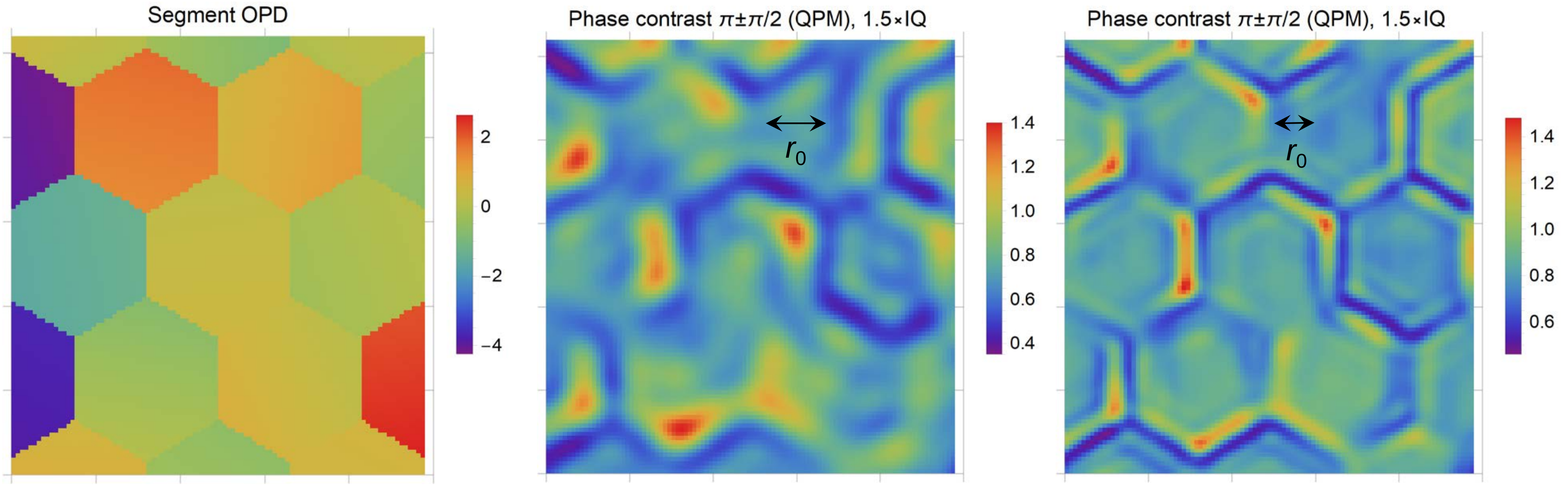
➔ *No good segment tip/tilt information from phase contrast sensor*

Phase Contrast WFS: Influence of Seeing

- 1780 nm (H-band), averaged over 4000 phase screens,
- Seeing: 0.67” vs. 1.1” at 500 nm, IQ: 0.37”/0.68”, r_0 : 602/367 mm

+ apodization pinhole:
Ø = 3.5 × IQ

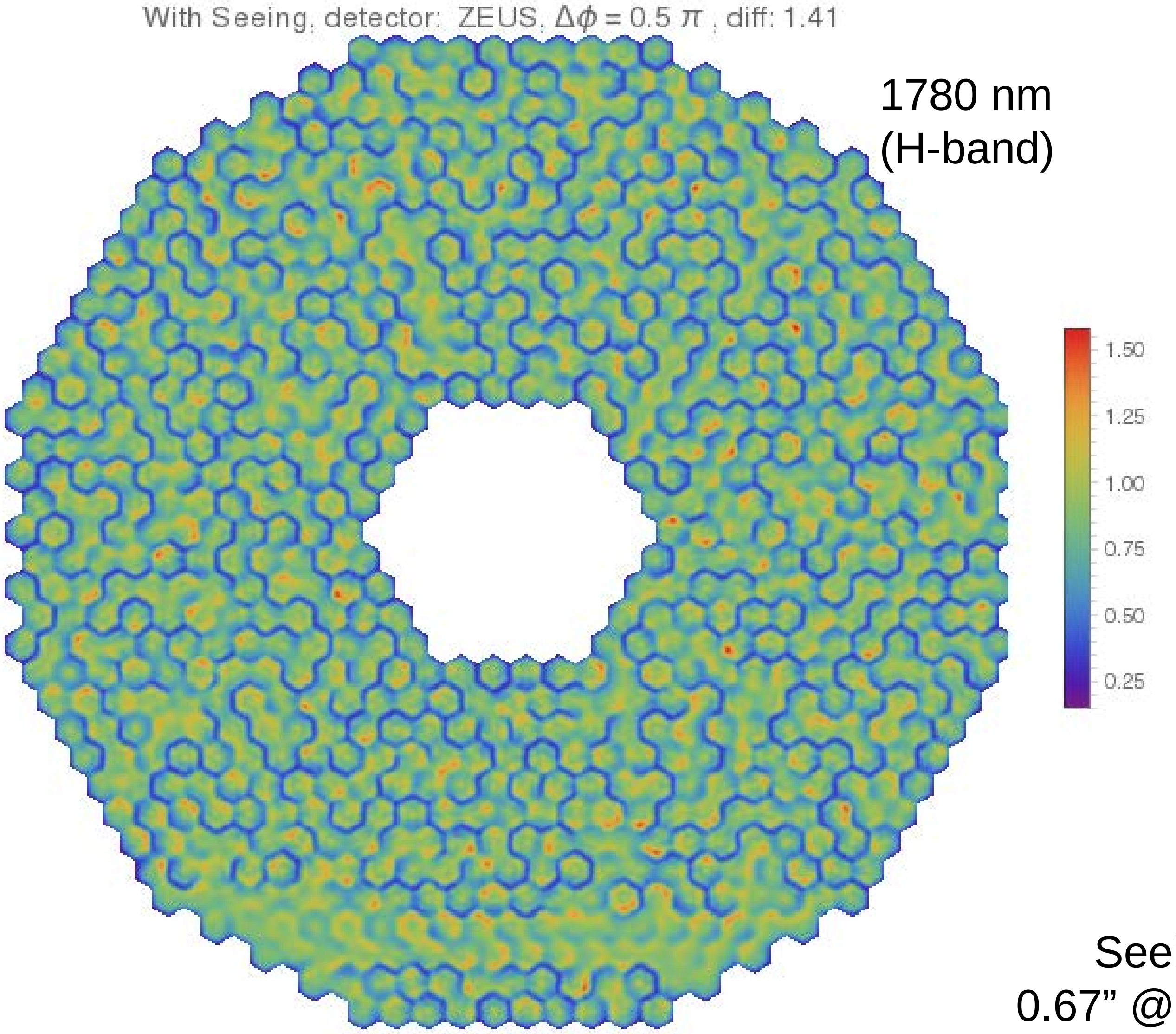




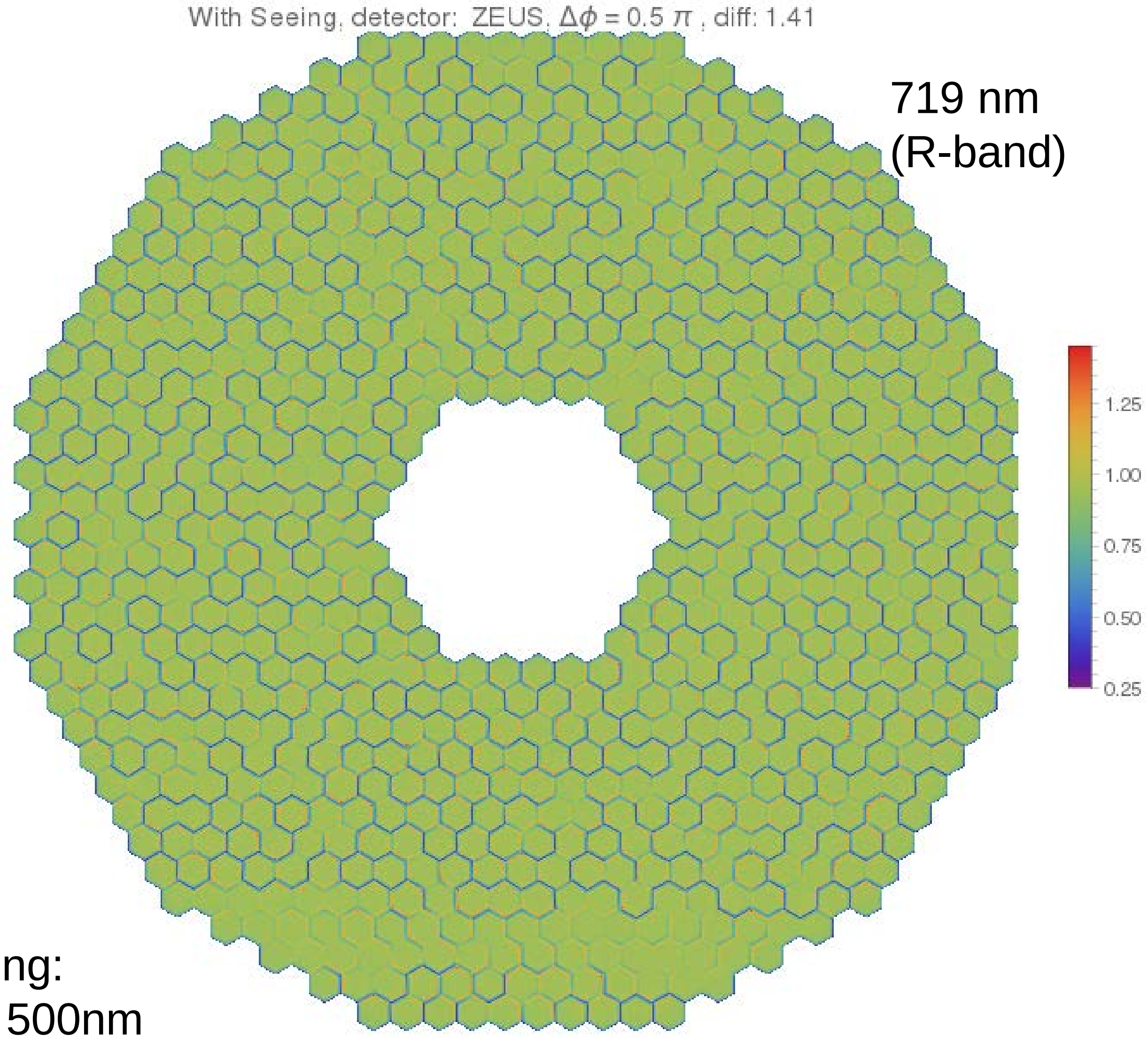
IQ 0.37”, r_0 : 602 mm

IQ 0.68”, r_0 : 367 mm

Phase Contrast Response Wavelength-Dep.



1780 nm
(H-band)

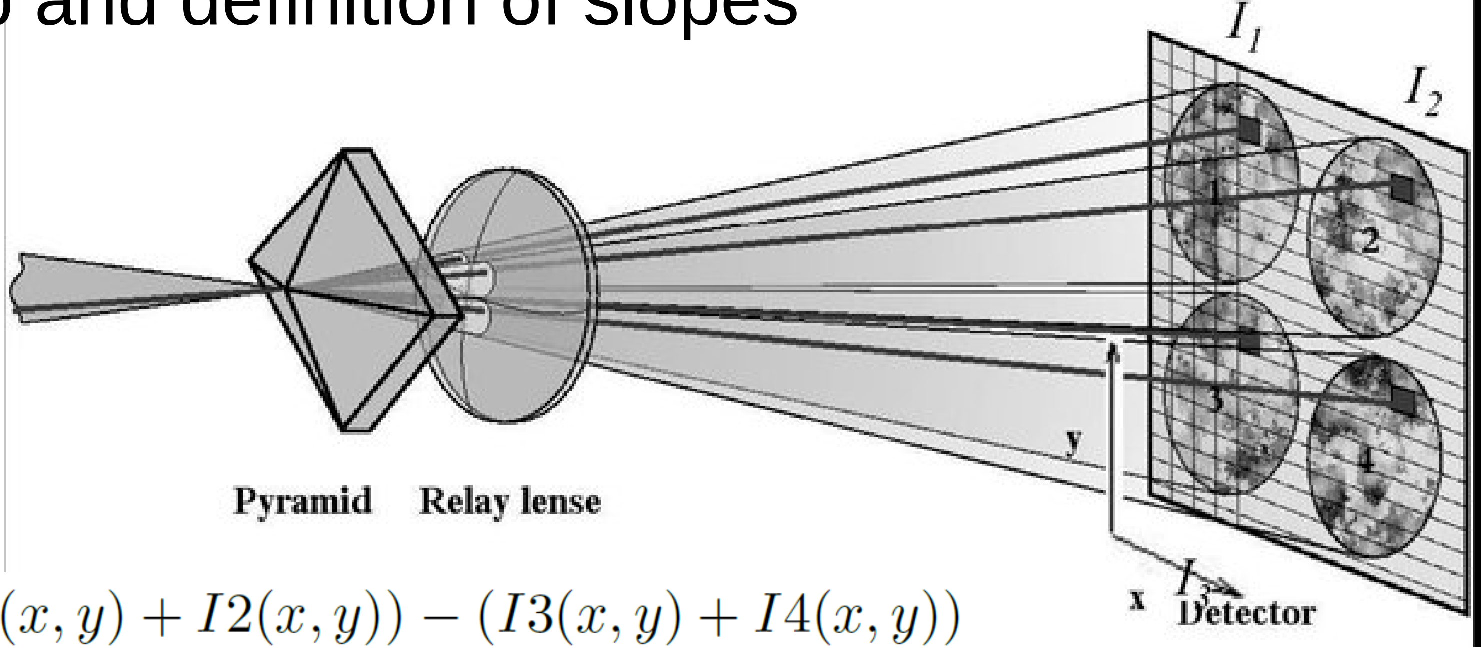


719 nm
(R-band)

Seeing:
0.67” @ 500nm

Pyramid WFS

Pyramid WFS setup and definition of slopes



$$Sx(x,y) = \frac{(I1(x,y) + I2(x,y)) - (I3(x,y) + I4(x,y))}{Iavg}$$

$$Sy(x,y) = \frac{(I1(x,y) + I4(x,y)) - (I2(x,y) + I3(x,y))}{Iavg}$$

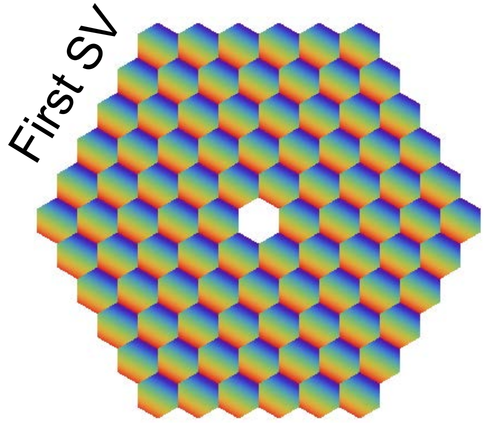
$$Iavg(x,y) = I1(x,y) + I2(x,y) + I3(x,y) + I4(x,y)$$

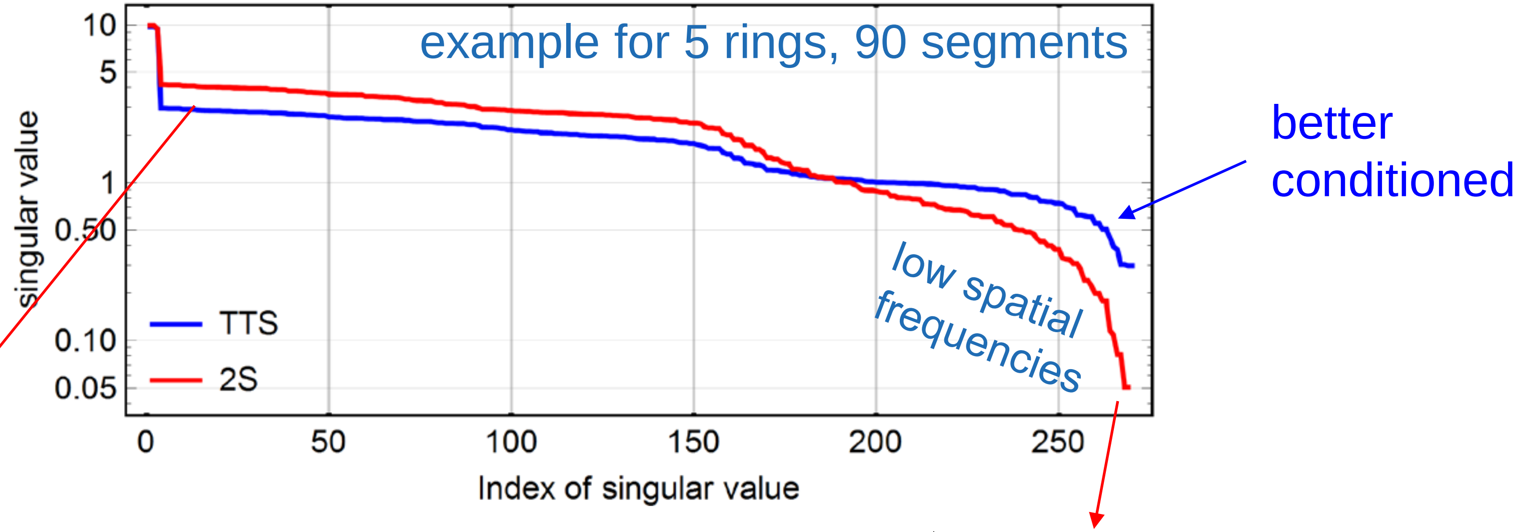
Source:
Lardière et al., 2017

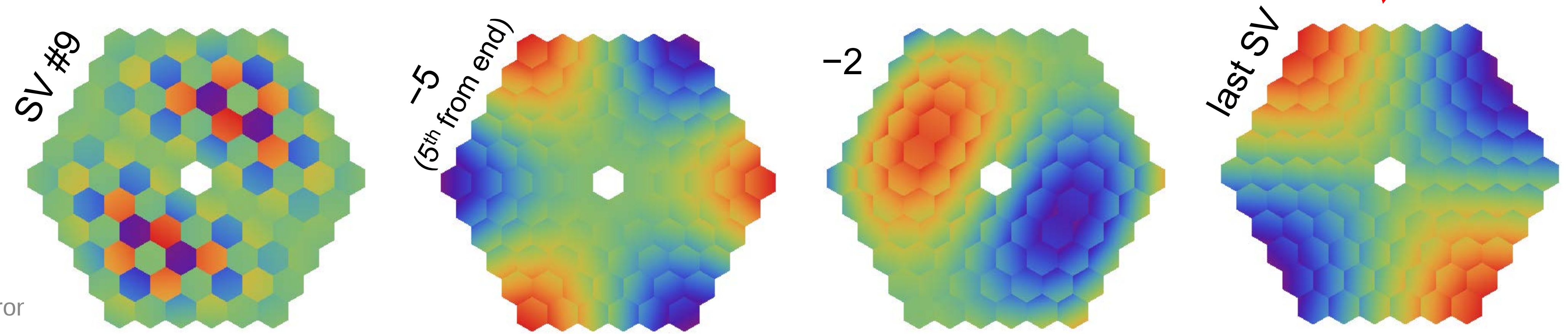
Jacob *et al.*, SPIE 107001 (2018)

Conditioning for 90 Segments

- The invertibility of the equation system is dictated by its conditioning number (ratio of largest to smallest singular value of matrix ***A***)

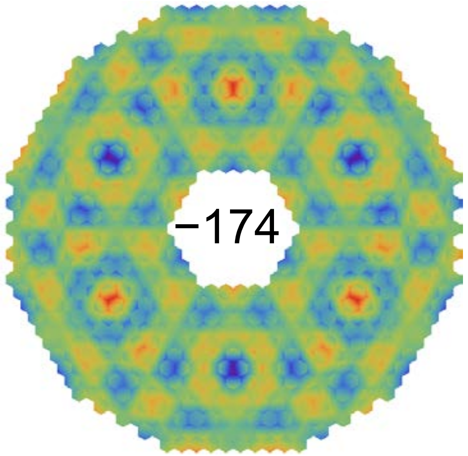


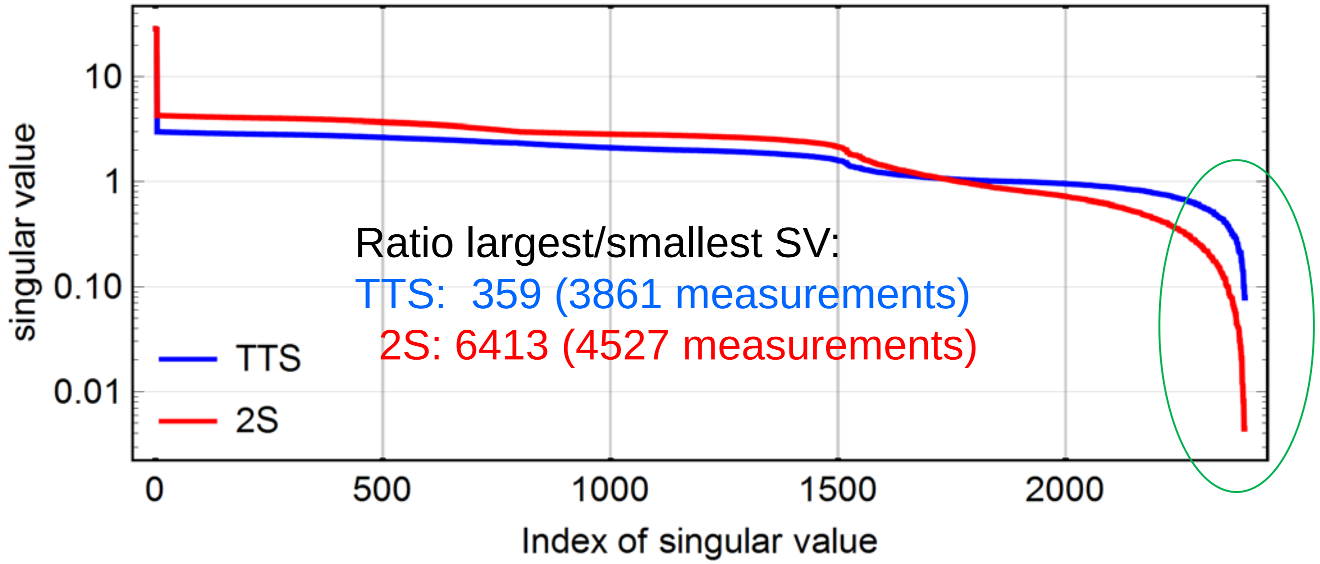


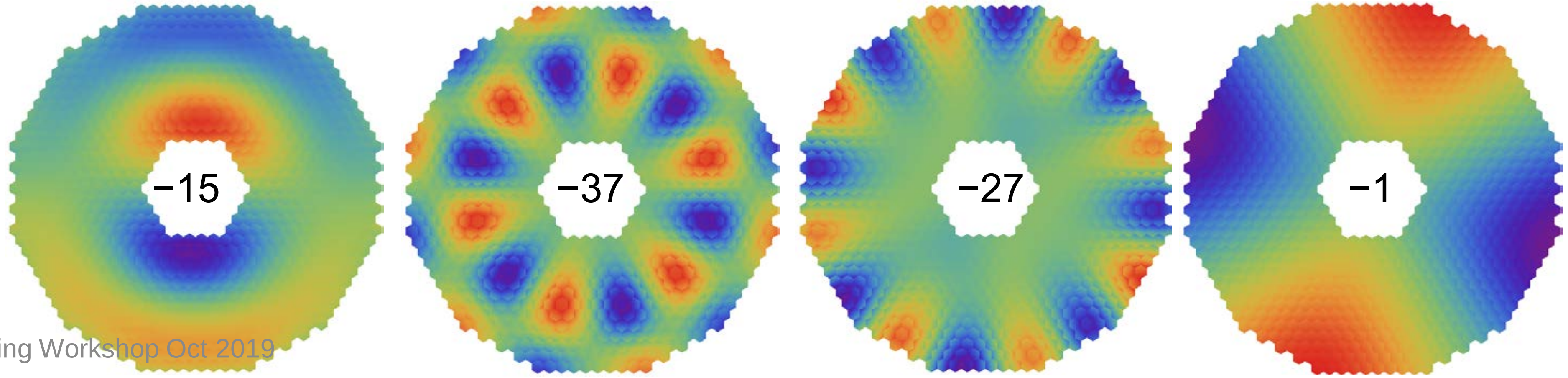


Conditioning for 798 Segments

- Conditioning difference TTS vs. 2S grows with the number of segments







Isabel's ZEUS step fit function

- From Isabel Surdej's Ph.D. thesis, p.86/87

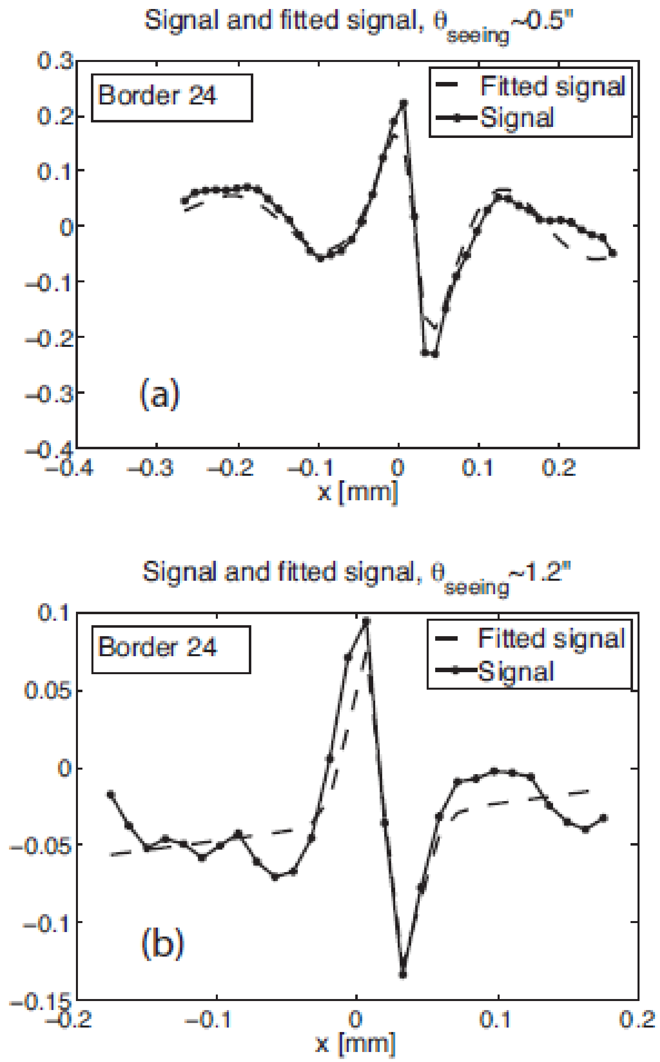
6.4 Fitting algorithm

The theoretically expected signal of Eq. (5.30) can be expressed as a function of the piston step and a few other nuisance parameters, which affect the signal but do not alter the information on the piston step. A theoretically derived function with a few free parameters is fitted in the least squares sense to the measured intensity profiles perpendicular to the segment edges.

Expressing the theoretical equation of the signal (Eq. (5.30)) as a function of the fitted parameters results in the following formula:

$$F(x) = a_3 + a_4(x - a_5) + [1 - f(u)]\{a_1 \mathrm{sign}(x - a_5)\sin(\psi_0) - (1 - a_2) f(u)(1 - \cos(\psi_0))\} \quad (6.10)$$

where $u = a_6|x - a_5|$. a_i, $(i = 1 \ldots 6)$, are six free fitting parameters. The first term a_3 represents a constant background, the second term a constant slope in the signal, a_5 represents the shift of the signal with respect to the origin and a_6 determines the signal width.



Isabel's ZEUS step fit function (II)

- From I. Surdej's Ph.D. thesis, p.87 (continued)

$f(u)$, (Eq. (3.25)), is the normalized sinc integral for the sharp edge mask (round pinhole):

$$f(u) = \frac{2}{\pi}\mathrm{Si}(u) = \frac{2}{\pi}\int_0^u \frac{\sin(t)}{t}\mathrm{d}t, \tag{6.11}$$

and for the gaussian pinhole (or in the presence of atmospheric turbulence) $f(u)$ is given by the Gaussian error function (Eq. (3.24)):

$$f(u) = \Phi(u) = \frac{2a_6}{\sqrt{\pi}}\int_0^u \exp(-t^2)\mathrm{d}t. \tag{6.12}$$

In the presence of atmospheric turbulence, a simplified model of the signal, as presented in Section 5.4.5, is used.

The piston step information is contained in the two parameters, a_1 and a_2, which are proportional to the sine and cosine of the phase $\Delta\varphi$, respectively,

$$\begin{cases} a_1 = C_1 \sin(\Delta\varphi) \\ a_2 = C_2 \cos(\Delta\varphi) \end{cases}. \tag{6.13}$$

Under ideal conditions, C_1 and C_2 are equal to 1, as described by Eq. (5.30). However, in the presence of noise such as atmospheric disturbances or polishing errors the values of C_1 and C_2 are modified and are smaller than 1, as explained in [86]. In the fitting, it is assumed that a_1 and a_2 are two independent parameters.